An optimal engineering design method with failure rate constraints and sensitivity analysis. Application to composite breakwaters

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Abstract

The paper introduces a new approach to composite breakwater design based on minimizing initial/construction costs subject to yearly failure rate bounds for all failure modes, and presents a technique for sensitivity analysis. The solution of the resulting optimization problem becomes complex because the evaluation of failure rates involves one optimization problem per failure mode (FORM), so that a decomposition method is used to solve the problem. In addition, a sensitivity analysis is performed, which makes it possible to determine how the cost and yearly failure rates of the optimal solution are affected by small changes in the input data values. The proposed method is illustrated by its application to the design of a composite wall under breaking and non-breaking wave conditions. The storms are assumed to be stochastic processes characterized by their maximum significant wave heights, their maximum wave heights and the associated zero-up-crossing mean periods.

Keywords: Cost optimization; Failure probability; Modes of failure; Stochastic process; Reliability analysis; Safety factors

1. Introduction

The phases that an engineering structure undergoes are: construction, service life and dismantling. In addition, maintenance and repair take place during the service lifetime. During each of these phases, the structure and the environment undergo a continuous sequence of outcomes, the consequences of which have to be considered in the project. The objective of the design is to verify that the structure satisfies the project requirements during these phases in terms of acceptable failure rates and cost (see Losada, 1990 and ROM, 2001).

Since repair depends on the modes of failure and their occurrence frequencies, these must be defined. A mode describes the form or mechanism in which the failure of the structure or one of its elements occurs. Each mode of failure is defined by a corresponding limit state equation as, for example:

\[ g_m(x_1, x_2, \ldots, x_n) = h_{am}(x_1, x_2, \ldots, x_n) \]

\[ - h_{fm}(x_1, x_2, \ldots, x_n); \ m \in M, \] (1)

where \( (x_1, x_2, \ldots, x_n) \) refer to the values of the variables involved, \( g_m(x_1, x_2, \ldots, x_n) \) is the safety margin and \( h_{am}(x_1, x_2, \ldots, x_n) \) and \( h_{fm}(x_1, x_2, \ldots, x_n) \) are two opposing magnitudes (such as stabilizing and mobilizing forces, strengths and stresses, etc.) that tend to prevent and produce the associated mode of failure, respectively, and \( M \) is the set of all failure modes.

In this paper it is supposed that failure occurs during storms that are assumed to be stochastic processes of random intensity, and that failure occurs when the critical variables (extreme wave heights and periods) satisfy \( g_m \leq 0 \). Then, the probability \( P_{fm} \) of failure mode \( m \) in a given period becomes:

\[ P_{fm} = \int_{g_m(x_1, x_2, \ldots, x_n) \leq 0} f_{x_1, x_2, \ldots, x_n}(x_1, x_2, \ldots, x_n) dx_1 dx_2 \ldots dx_n, \] (2)

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where \( f_{X_1, X_2, \ldots, X_n} (x_1, x_2, \ldots, x_n) \) is the joint probability density function of all variables involved in the problem. With this information, and the consideration of all storms that may occur in a year, the different yearly failure rates for all failure modes can be estimated.

If the design variables lead to admissible failure rates, i.e., below given upper bounds, the design is said to be safe. The main advantage of probabilistic based design is that the reliability of the structure can be evaluated. However, they are very sensitive to tail assumptions (behavior of the random variables for extreme values) (see Galambos, 1987; Castillo, 1988), and in some cases, as, for example, vertical wall stability, run-up, overtopping, geotechnical stability, etc., the dependence structure and the statistical distributions of the variables involved are difficult to define.

Over the last few years, design methods have been improved by applying optimization techniques. The main advantage is that these techniques lead to optimal design and automation, i.e., the values of the design variables are provided by the optimization procedure (the optimal values) and not fixed by the engineer. Designer concerns are the constraints to be imposed on the problem and the objective function to be optimized.

Some authors consider the construction cost (Castillo et al., 2003a,b,c,d, 2004) or the total cost (construction, maintenance and repairs) as the design criteria (Van Dantzig, 1956; Voortman et al., 1998; Enevoldsen, 1991; Enevoldsen and Sorensen, 1993, 1994; Minguez et al., in press). As one of the main purposes of the different maritime structures is to protect harbor areas from being flooded by large waves, and because they can be used in very different conditions where the consequences of a partial or complete failure are also very different, the accepted corresponding probability of failure varies considerably. However, people should not allow engineers and politicians to make their decisions based only on economic criteria. Human life, quality and service reliability, and perhaps other criteria must be considered. In fact, some constraints on the yearly failure probability rate should be imposed. The evaluation of composite breakwater reliability implies solving as many optimization problems as failure modes. Thus, use of optimization programs is not straightforward.

In some cases (see Nielsen and Burcharth, 1983) cost evaluations take into account the occurrence of failures, but taking into account the actual sequence of failures is difficult. Large storms produce at most one single failure of each type (mode) or combinations of them, because even though several of its waves (the largest) are able to produce failure, once destroyed, the breakwater cannot be destroyed again before its repair, which will take place once the storm has finished.\(^1\) An evaluation of the number of failures must take into consideration that several dangerous sea waves normally occur during the same storm, but produce at most one failure of each type.

This implies that the natural event to predict the number of failures is the storm occurrence.

In addition to requiring optimal solutions to problems, some interest is shown by people in knowing how sensitive the solutions to data values are. A sensitivity analysis provides excellent information on the extent to which a small change in the parameters or assumptions (data) modifies the resulting design (geometric dimensions, costs, reliabilities, etc.). This will be useful to: (a) the designer in order to know how sensitive the design is to the assumptions, (b) the construction engineer to know to what extent changes in the unit prices and other data modify the cost and reliabilities, and (c) the code designer to know, for example, how much a lowering of the failure rate bounds increases the cost.

Though in the literature there are efficient one-level and two-level optimization techniques for reliability-based optimization problems, see e.g. Kuschel and Rackwitz (1997, 2000), Sorensen et al. (1994b), one needs methods able to deal with failure rates and sensitivity analysis.

The aims of this paper are: (a) to introduce a new approach of composite breakwater design based on minimizing initial/construction cost subject to yearly failure rate bounds for all failure modes, and (b) to present a technique for sensitivity analysis.

The paper is structured as follows. In Section 2 the probabilistic design is described. In Section 3 the proposed method for optimal design is presented. Section 4 illustrates the proposed method by an example application dealing with the design of a composite breakwater. Section 5 is devoted to the discussion of the statistical assumptions. Section 6 presents a numerical example. Finally, Section 7 gives some conclusions.

2. The probabilistic design problem: safe and failure domains

In the design and reliability analysis of a maritime structure, there are some random variables \((X_1, \ldots, X_n)\) involved. They include geometric variables, material properties, loads, etc. In this paper, without loss of generality, we make no distinction between random and deterministic variables. So, it is assumed that all variables involved are random, and deterministic variables are only particular cases of them. They belong to an \(n\)-dimensional space, which, for each mode of failure, can be divided into two domains, the safe and the failure domains:

\[
\begin{align*}
\text{Safe : } S &= \{ (x_1, x_2, \ldots, x_n) \mid g_m(x_1, x_2, \ldots, x_n) > 0 \} ; m \in M \\
\text{Failure : } F &= \{ (x_1, x_2, \ldots, x_n) \mid g_m(x_1, x_2, \ldots, x_n) \leq 0 \} \}
\end{align*}
\]

where \( M \) is the set of all modes of failure \( m \).

It is important to distinguish between design values of the random variables \(X_i\), and their actual values \(x_i\) \((i = 1, 2, \ldots, n)\). The design values are those values selected by the engineer at the design stage for the geometric variables (dimensions), the material properties (strengths, stiffness, etc.), that do not necessarily correspond with those in the real work. Thus, in this paper the design values are assumed to be the means or

\(^1\) The case of the sliding of a caisson, which can occur many times bit by bit during a single big storm for the sake of simplicity is assumed to occur here in one go.
the characteristic values (extreme percentiles) of the corresponding random variables, and are denoted \( \bar{x}_i \) (mean) and \( x_i \) (characteristic), respectively. Some of these design values are chosen by the engineer or given by the design codes, and some (associated with the design variables) are selected by the optimization procedure to be presented. In this paper, the set of variables \( (X_1, \ldots, X_n) \) will be partitioned into four sets (for the particular example of the composite breakwater see Appendix A):

1. Optimized design variables \( d \): Design random variables the mean values of which are to be chosen by the optimization procedure to optimize the objective function (minimize the initial-construction cost). Normally, they describe the dimensions of the work being designed, such as width, thickness, height, cross sections, etc., but can include material properties, etc.

2. Non-optimized design variables \( \eta \): Set of variables the mean or characteristic values of which are fixed by the engineer or the code guidelines as input data to the optimization program. Some examples are costs, material properties (unit weights, strength, Young modulus, etc.), and other geometric dimensions of the work being designed (parapet breakwater width, etc.) that are fixed.

3. Random model parameters \( \kappa \): Set of parameters used in the probabilistic design, defining the random variability and dependence structure of the variables involved (standard deviations, variation coefficients, correlations, etc.).

4. Dependent or non-basic variables \( \psi \): Dependent variables which can be written in terms of the basic variables \( d \) and \( \eta \) to facilitate the calculations and the statement of the problem constraints.

The corresponding means of \( d \) will be denoted \( \bar{d} \), and the mean or the characteristic values of \( \eta \) are denoted \( \bar{\eta} \).

The cost optimization problem to be stated in Section 3 will make use of these sets of variables.

Given a set of values of the design variables \( \bar{d} \), the probability of failure \( p^m_d \) under mode \( m \) during a random storm can be calculated using the joint probability density function \( f(x) = f_{X_1, X_2, \ldots, X_n}(x_1, x_2, \ldots, x_n; \Theta) \) of all variables involved, where \( \Theta \) is a parametric vector, by means of the integral:

\[
p^m_d(\Theta) = \int_{g_{m}(x_1, x_2, \ldots, x_n) \leq 0} f_{X_1, X_2, \ldots, X_n}(x_1, x_2, \ldots, x_n; \Theta) dx_1 dx_2 \ldots dx_n. \tag{4}
\]

In this paper we assume that the parametric vector \( \Theta = (\bar{d}, \bar{\eta}, \kappa) \) contains the means \( \bar{d} \), the means or the characteristic values \( \bar{\eta} \), and some other extra vector of random model parameters \( \kappa \).


Unfortunately, calculation of \( p^m_d(\Theta) \) is difficult. So, to eliminate the need for complex numerical integrations, the “First Order Reliability Methods” (FORM) transform the initial set of variables into an independent multinormal set and use a linear approximation. For a complete description of some of these methods and some illustrative examples see Hasofer and Lind (1974), Madsen et al. (1986), Ditlevsen and Madsen (1996), or Melchers (1999).

In this paper we use first order reliability methods (FORM) for evaluating \( p^m_d(\bar{d}, \bar{\eta}, \kappa) \) for \( m = 1, 2, \ldots, M \). In FORM the involving random variables \( X \) are transformed into standard unit normals \( Z \) and the limit state boundaries approximated by hyperplanes, so that the reliability indices can be easily obtained by solving the following optimization problem

\[
p^m_d(\Theta) = \text{Maximum } \Phi(-\beta_m) = \Phi(-\sqrt{\gamma^T \gamma}), \tag{5}
\]

i.e., maximizing with respect to \( z \), subject to

\[
g_{m}(T(z, \Theta), \Theta) = 0, \tag{6}
\]

where \( X = T(Z, \Theta) \) denotes the transformation from standard normal stochastic variables \( Z \) to basic variables \( X \), \( \beta_m \) is the reliability index for failure mode \( m \), and \( \Phi(\cdot) \) is the cumulative distribution function of the standard normal random variable.

Note that we do not minimize \( \beta_m \) in Eq. (5) as usual, but maximize the probability of failure \( \Phi(-\sqrt{\gamma^T z}) \). However, since the functions \( \Phi(\cdot) \) and square root are increasing in their arguments, both approaches are equivalent. The function \( \Phi(-\sqrt{\gamma^T z}) \) has been chosen because we later look for the probability of failure sensitivities with respect to the data, i.e., the rate of change of \( \Phi(-\beta_m) \) with respect to the data values.

3. Proposed method for optimal design

To design the maritime structure we propose to minimize the initial/construction cost subject to failure rate constraints. Since the latter involves random occurrences, some model assumptions are necessary. Note that contrary to the material in Section 2, which is well known by reliability people, some of the formulas and the model to be presented in Sections 3 and Appendices E and F are original.

3.1. Model assumptions

Before describing the model assumptions, it is worthwhile mentioning that the analysis of the composite breakwater example to be discussed below cannot be considered exhaustive, because, for example, several failure modes were not implemented (settlement, scour, deterioration and corrosion of reinforcement due to chloride ingress through the concrete or concrete cracks, etc.) and some hydraulic responses were not analyzed (wave transmission, wave reflection).

Our model is based on the following assumptions:

1. The storms are assumed to be stochastic processes, i.e., to occur at random times with a yearly rate \( \nu_n \) (mean number of storms per year). Note that no assumption is needed about the
dependence or independence of storms or the distribution of occurrence times, because only the yearly failure rate is sought.

(2) Long-term statistics deal with the distribution of the storms which are characterized by a set of three variables that represent the maximum significant wave height $H_{\text{max}}$ during each storm, its maximum wave height $H_{\text{max}}$, and the associated wave period $T_{\text{max}}$ (that occurring with $H_{\text{max}}$). It is assumed that they are dependent random variables whose probability distribution and dependence structure must be derived from real data. Once a storm has occurred, its intensity and characteristics can be derived from this joint distribution, i.e., a set of values $(H_{\text{max}}, H_{\text{max}}, T_{\text{max}})$ can be drawn at random from a population with the corresponding distribution. For the sake of simplicity, we assume that these variables provide enough information to verify the breakwater failure modes.

(3) Failures occur during storms and the probability of failure in mode $m$ in a random storm is $p_{\text{st}}^m(\bar{d}, \bar{\eta}, \kappa)$, which has been considered to be a function of the design variables and parameters $(\bar{d}, \bar{\eta}, \kappa)$, which include the geometric dimensions of the breakwater and the parameters defining the probability distribution of all variables involved.

(4) One storm can cause at most only one failure of each type (mode), because in the case of the occurrence of several sea waves in one storm all able to produce failure, only the first failure of each mode must be considered, because repair is not possible during storms. This implies, as indicated in the introduction, that failure accumulation is not included.

(5) A failure mode does not induce any other failure modes. This means that the structure is assumed not to suffer a progressive collapse. However, different failure modes can occur simultaneously, and they are not statistically independent because they have common inducing agents. Interaction between failure modes is an important problem. However, we have to bear in mind that nowadays there is not enough knowledge on such interaction for it to be included in models; we are still trying to understand and to evaluate how individual modes of failure start and progress. Thus, to complicate the presentation of a new optimization procedure with additional heuristic approaches is, in the authors’ opinion, not the best decision for this paper, though, for example, a model for interaction between the toe berm and the main armour for rubble mound breakwaters is presented in Christiani (1997).

(6) The occurrence of failure in mode $m$, with probability $p_{\text{st}}^m(\bar{d}, \bar{\eta}, \kappa)$, is a Bernoulli random variable, and the mean number of storms per year is $r_m$. Thus, the mean number of failures per year (yearly failure rate) is equal to the mean of the Binomial random variable $B(r_{\text{st}}, p_{\text{st}}^m)$:

$$r_m = r_{\text{st}} p_{\text{st}}^m(\bar{d}, \bar{\eta}, \kappa).$$

Note that the number of failures is a Binomial random variable $B(r_{\text{st}}, p_{\text{st}}^m)$. Alternatively, the Poisson distribution $P(r_{\text{st}} p_{\text{st}}^m)$ could be used as a very good approximation to the Binomial distribution if $p_{\text{st}}^m \leq 1$ and $r_{\text{st}} p_{\text{st}}^m \leq 5$.

(7) The proposed approach is based on guaranteeing bounded yearly failure rates of all failure modes. However, for the global failure rate, one can consider the well known bounds:

Lower bound : $P_f = \max_m r_m^m$; \hspace{1cm} (8)

Upper bound : $P_f = 1 - \prod_{m=1}^M (1 - r_m)$,

where $P_f$ is the global failure rate (upper bound failure rates could be included in the proposed method without additional effort).

Note that the failure rates (7) can be immediately transformed into probabilities associated with the lifetime of the structure ($D$) using the following expression:

$$P_f^D = 1 - (1 - r_m)^D.$$ \hspace{1cm} (9)

The reason for using failure rates as an alternative to lifetime failure probability is because, in our opinion, this makes more sense and allows a comparison of the reliability of different designs made for different lifetimes.

3.2. Initial/construction cost function

In this paper the criteria for design are based on minimizing the initial or construction costs subject to bounded yearly failure rates, i.e., it is assumed that politicians behave under short-term policies, but they are limited by some reliability constraints. In this paper we do not consider the cost of damage due to serviceability limit states and to ultimate limit states (see Sorensen et al., 1994a; Voortman et al., 1998), but these costs can be bounded using the yearly failure rate bounds. Thus, the initial/construction cost ($C_{\text{ca}}(\bar{d}, \bar{\eta})$), is given by:

$$C_{\text{ca}}(\bar{d}, \bar{\eta}) = C_{\text{ca},V} + C_{\text{sp},V} + C_{\text{sd},V} + C_{\text{co},V},$$

where $\bar{d}$ and $\bar{\eta}$ are the corresponding mean values of characteristic variables, respectively, $V_{\text{ca}}, V_{\text{sp}}, V_{\text{sd}}$ and $V_{\text{co}}$ are the caissons, superstructure, armor layer, and core volumes, respectively, and $C_{\text{ca},V}, C_{\text{sp},V}, C_{\text{sd},V}$ and $C_{\text{co},V}$ are the respective construction costs per unit volume in Spain. Note that the sand-filled caisson cost ($C_{\text{ca}}$) includes the material cost of 80% of sand fill, 20% of structural concrete, and construction, launching, transport and sinking. The details of the derivation of the cost function are given in Appendix B.

3.3. Evaluation of the failure mode probabilities in a random storm

In this paper we evaluate the failure mode probabilities in a random storm using first order reliability methods (FORM). More precisely, $p_{\text{st}}^m(\bar{d}, \bar{\eta}, \kappa)$ for $m=1, 2, \ldots, M$ is obtained using Eqs. (5) and (6).

Once the probabilities for all failure rates have been calculated it is possible to obtain the yearly failure rates for all modes. Thus, once the failure rate bounds are decided, their incorporation into the optimization procedures as additional constraints can be done as follows:

$$r_m(\bar{d}, \bar{\eta}, \kappa) = r_{\text{st}} p_{\text{st}}^m(\bar{d}, \bar{\eta}, \kappa) \leq R_m^D; \hspace{1cm} m=1, 2, \ldots, M.$$

$$r_m(\bar{d}, \bar{\eta}, \kappa) = r_{\text{st}} p_{\text{st}}^m(\bar{d}, \bar{\eta}, \kappa) \leq R_m^D; \hspace{1cm} m=1, 2, \ldots, M.$$ 

$$r_m(\bar{d}, \bar{\eta}, \kappa) = r_{\text{st}} p_{\text{st}}^m(\bar{d}, \bar{\eta}, \kappa) \leq R_m^D; \hspace{1cm} m=1, 2, \ldots, M.$$ 

$$r_m(\bar{d}, \bar{\eta}, \kappa) = r_{\text{st}} p_{\text{st}}^m(\bar{d}, \bar{\eta}, \kappa) \leq R_m^D; \hspace{1cm} m=1, 2, \ldots, M.$$ 

$$r_m(\bar{d}, \bar{\eta}, \kappa) = r_{\text{st}} p_{\text{st}}^m(\bar{d}, \bar{\eta}, \kappa) \leq R_m^D; \hspace{1cm} m=1, 2, \ldots, M.$$ 

$$r_m(\bar{d}, \bar{\eta}, \kappa) = r_{\text{st}} p_{\text{st}}^m(\bar{d}, \bar{\eta}, \kappa) \leq R_m^D; \hspace{1cm} m=1, 2, \ldots, M.$$ 

$$r_m(\bar{d}, \bar{\eta}, \kappa) = r_{\text{st}} p_{\text{st}}^m(\bar{d}, \bar{\eta}, \kappa) \leq R_m^D; \hspace{1cm} m=1, 2, \ldots, M.$$ 

$$r_m(\bar{d}, \bar{\eta}, \kappa) = r_{\text{st}} p_{\text{st}}^m(\bar{d}, \bar{\eta}, \kappa) \leq R_m^D; \hspace{1cm} m=1, 2, \ldots, M.$$ 

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$$r_m(\bar{d}, \bar{\eta}, \kappa) = r_{\text{st}} p_{\text{st}}^m(\bar{d}, \bar{\eta}, \kappa) \leq R_m^D; \hspace{1cm} m=1, 2, \ldots, M.$$ 

$$r_m(\bar{d}, \bar{\eta}, \kappa) = r_{\text{st}} p_{\text{st}}^m(\bar{d}, \bar{\eta}, \kappa) \leq R_m^D; \hspace{1cm} m=1, 2, \ldots, M.$$ 

$$r_m(\bar{d}, \bar{\eta}, \kappa) = r_{\text{st}} p_{\text{st}}^m(\bar{d}, \bar{\eta}, \kappa) \leq R_m^D; \hspace{1cm} m=1, 2, \ldots, M.$$ 

$$r_m(\bar{d}, \bar{\eta}, \kappa) = r_{\text{st}} p_{\text{st}}^m(\bar{d}, \bar{\eta}, \kappa) \leq R_m^D; \hspace{1cm} m=1, 2, \ldots, M.$$ 

$$r_m(\bar{d}, \bar{\eta}, \kappa) = r_{\text{st}} p_{\text{st}}^m(\bar{d}, \bar{\eta}, \kappa) \leq R_m^D; \hspace{1cm} m=1, 2, \ldots, M.$$ 

$$r_m(\bar{d}, \bar{\eta}, \kappa) = r_{\text{st}} p_{\text{st}}^m(\bar{d}, \bar{\eta}, \kappa) \leq R_m^D; \hspace{1cm} m=1, 2, \ldots, M.$$
where $R_m^0$ is the corresponding failure rate upper bound for failure mode $m$, which should be fixed by the codes.

Everything is now ready to state the design problem as an optimization problem as follows.

3.4. Design as an optimization problem

In this paper the design of a maritime structure is equivalent to solving the following optimization problem:

$$\text{Minimize } C_0(d, \tilde{h}),$$

subject to the yearly failure rate constraints, the equations that allow obtaining the intermediate variables, and the geometric constraints:

$$r_m(d, \tilde{h}, \kappa) = r_{st} p_m^m(d, \tilde{h}, \kappa) \leq R_m^0; \quad m = 1, \ldots, M,$$

$$q(d, \tilde{h}) = \psi,$$

$$h(d, \tilde{h}, \psi) \leq 0,$$

where $p_m^m(d, \tilde{h}, \kappa)$ is given by solving the problem (5) and (6).

The constraints (13) are called complicating constraints, because they involve inner optimization problems.

In Appendix E we explain how the optimization problem in Eqs. (12)–(15) can be solved using decomposition techniques.

In Appendix F we also give some methods to calculate the sensitivities of the costs and reliabilities to the model parameters.

4. Optimized design of a composite breakwater

The probability-based design of composite breakwaters has been studied by Christiani et al. (1996), Burcharth and Sorensen (1998), Sorensen and Burcharth (2000), as well as in the European project PROVERBS (see Oumeraci et al., 2001) and the Working Group 28 on Breakwaters with Vertical and Inclined Concrete Walls PIANC (2003).

In this section the proposed procedure is applied to the design of a composite breakwater. The main section of the breakwater is shown in Fig. 1 where the main parameters are shown. Notice that these parameters define geometrically the different elements of the cross section and must be defined in the construction drawings. Our goal is an optimal design.

4.1. Modes of failure

In this study a total of 7 modes of failure have been considered: sliding failure ($s$), 4 foundation failures ($b, c, d, rs$), overtopping failure ($o$), and seaside berm instability failure ($a$), as shown in Fig. 2. But other failure modes, such as settlement, scour, deterioration and corrosion of reinforcement due to chloride ingress through the concrete or concrete cracks, wave transmission, wave reflection, etc. could have been considered.

As the consequences of failure can be very different depending on the type of the structure and the environment, the limit states can be classified as:

1. Operating suspension. It consists of the disruption of normal use of the structure, usually produced by atmospheric agents, such as storms, hurricanes, etc. In these situations the safety of using the structure decreases considerably but the structure recovers the normal use once the external agents disappear. This limit state is widely used in harbors, because ships cannot operate during sea storms. In this paper the overtopping failure ($o$) is included in this set.

2. Damage. Situations where important repairs are required to prevent the failure or collapse of the structure, these include, for example, excessive or premature cracking, deformation or permanent inelastic deformation, etc. Seaside berm instability failure ($a$) belongs to this group, because, although its failure does not imply the destruction of the breakwater, major reparations are required.

3. Ultimate. These limit states refer to the collapse of all or part of the structure, such as tipping or sliding, rupture, progressive collapse, plastic mechanism, instability, corrosion, fatigue, deterioration, fire, etc. Damage limit states often are included in the category. Since the ultimate limit state failure of the composite breakwaters is determined by subsoil failure, as pointed out by

![Fig. 1. Composite breakwater showing the geometric design variables.](image-url)
Voortman et al. (1998), and Martinelli et al. (1999), then, the sliding (s), and the 4 foundation failures (b, c, d, rs) are included in this set.

Some of these modes are correlated, because they have common agents, or because one mode can induce the occurrence of others. Only the correlation due to common agents is considered in this paper.

The external wave forces on the upright section are the most important considerations in the design of vertical breakwaters, including both pulsating and impact wave loads. The well known Goda pressure formulas (see Goda, 1985) for the evaluation of the forces acting on the breakwater have been used in this paper. Yet, as the impulsive pressure coefficient used in Goda's formula does not accurately estimate the effective pressure due to impulsive pressure under all condi-
tions, the new impulsive pressure coefficient proposed by Takahashi et al. (1992) is used. The maximum wave height \( H_{\text{max}} \) is adjusted in the surf zone due to random wave breaking as described by Goda (1985):

\[
\frac{H_{\text{max}}}{L_0} \leq A \left\{ 1 - \exp \left( -1.5 \frac{\pi h_0}{L_0} \left( 1 + 15 \tan^{4/3} \theta_b \right) \right) \right\}, \tag{16}
\]

where \( h_0 \) is the water height in the distance of five times the maximum significant wave height \( H_{\text{max}} \), toward the offshore of the breakwater, \( L_0 \) is the deep water wave length, \( \theta_b \) is the angle of the sea bottom and the coefficient \( A \) takes different values depending of the type of waves, for example, it takes the value 0.17 for regular waves. Its lower and upper limits are 0.12 and 0.18, respectively.

Thus the design wave height \( H_d \) is:

\[
H_d = \min(H_{\text{max}}, H_{\text{break}}), \tag{17}
\]

where \( H_{\text{break}} \) is the maximum wave height by breaking conditions obtained from Eq. (16).

### 4.1.1. Sliding failure

This failure occurs when the breakwater caisson suffers horizontal displacement. It can occur as a slip either at the interface between the caisson concrete base and the rubble material, or entirely in the rubble material. Safety against sliding failure can be verified by the following limit state equation (see Fig. 2(b)):

\[
g_s = \min(\mu_c, \tan \phi_e)(W_1 - F_v) - F_h, \tag{18}
\]

where \( \mu_c \) is the friction coefficient between the caisson and the rubble bedding layer, \( \phi_e \) is the reduced effective friction angle of the rubble mound, \( W_1 \) is the actual caisson weight reduced for buoyancy, and \( F_h \) and \( F_v \) are the total horizontal and vertical forces due to wave pressure, which are given by:

\[
W_1 = \gamma_c B h_s + \gamma_{sp} (B h_b + w_o h_o) - h' B \gamma_w, \tag{19}
\]

\[
F_h = h_c (p_1 + p_4)/2 + h' (p_1 + p_3)/2, \tag{20}
\]

\[
F_v = \frac{1}{2} p_o B, \tag{21}
\]

where \( \gamma_c \) is the average unit weight of the sand-filled caisson, \( B \) the caisson width, \( h_s \) is the sand-filled caisson height, \( \gamma_{sp} \) is the average unit weight of the concrete superstructure, \( h_b \) is the concrete crown height, \( h_o \) and \( w_o \) are the parapet breakwater height and width, respectively, \( h' \) is the submerged height of the caisson, \( \gamma_w \) is the water unit weight, \( h_c \) is the freeboard, \( p_1, p_3 \) and \( p_4 \) are the Goda pressures at water level, \( h_c \) of the bottom and freeboard, respectively, and \( p_o \) is the uplift pressure.

Note that the reduced effective friction angle of the rubble is given by the formula:

\[
\tan \phi_e = \frac{\sin \phi \cos \psi_e}{1 - \sin \phi \sin \psi_e}, \tag{22}
\]

where \( \phi \) is the effective friction angle of the rubble and \( \psi_e \) is the dilation angle.

### 4.1.2. Foundation failure

The following geotechnical failure functions estimated using the upper bound theorem of classical plasticity theory (see Sorensen and Burchard, 2000 or Oumeraci et al., 2001) are used in this paper:

1. Rupture surface through rubble only (rotation failure) (b).
2. Rupture surface through rubble only (straight rupture line) (c).
3. Rupture in rubble and sliding along top of subsoil (d).
4. Rupture in rubble mound and sand subsoil (rs).

It is often very practical to consider the equilibrium of the caisson separately from the equilibrium of the soil, thus the integrated effective stresses acting on the skeleton of rubble foundation are obtained as resultant from the other forces acting on the caisson. The distance of the vertical force \( W_1 - F_v \) component to the harbor side edge \( B_z \) is:

\[
B_z = 2 \frac{W_1(y - F_v) y_{F_h} - F_v y_{F_v}}{W_1 - F_v}, \tag{23}
\]

where \( y = M_1 / W_1 \), \( y_{F_h} = M_{F_h} / F_h \) and \( y_{F_v} = M_{F_v} / F_v \) are the lever arms of \( W_1 \), \( F_h \) and \( F_v \), respectively, and \( M_1, M_{F_h} \) and \( M_{F_v} \) are the corresponding moments given by:

\[
M_1 = \gamma_c h_s B^2 / 2 + \gamma_{sp} (h_b B^2 / 2 + w_o h_o (B - w_o / 2)) - \gamma_c h' B^2 / 2, \tag{24}
\]

\[
M_{F_h} = - \frac{2}{3} F_h B = \frac{1}{3} p_o B^2, \tag{25}
\]

\[
M_{F_v} = \frac{1}{6} (2p_1 + p_3) h' h_c^2 + \frac{1}{2} (p_1 + p_4) h' h_c + \frac{1}{6} (p_1 + 2p_4) (h_c)^2. \tag{26}
\]

The effect of wave induced pressure along the rupture boundary inside the rubble \( F_{hu} \) can be obtained under the assumptions of triangular pressure distribution in the horizontal direction and hydrostatic pressure in the vertical direction as:

\[
F_{hu} = \begin{cases} 
\frac{B_2^2 \tan \theta_s}{B} p_o & \text{if } B_x \leq h_s / \tan \theta_s, \\
\frac{h_o (2B_x - h_s / \tan \theta_s)}{2B} p_o & \text{if } B_x > h_s / \tan \theta_s,
\end{cases} \tag{27}
\]

where \( h_o \) is the core height, and \( \theta_s \) is the angle between the bottom of the wall and the rupture surface (see Fig. 2 (d), (e) and (f)).

The safety against rotation failure can be verified by the following limit state equation (see Fig. 2 (c)):

\[
g_s = B_z^2 (\gamma_s - \gamma_w) \tan \phi_e (\tan^2 (\pi / 4 + \phi_e / 2) \exp (\pi \tan \phi_e) - 1) - (W_1 - F_v) \left( \frac{1}{1 - F_h / (W_1 - F_v)} \right)^3, \tag{28}
\]

where \( \gamma_s \) is the rubble mound unit weight.
The failure mechanism through rubble only consists of a unit displacement along the line AB and is described by the angle $\theta_e$. The area of zone 1, including the leeward armor layer, can be written as:

$$A_1 = \frac{1}{4} \left( B_z + b + \frac{e}{ \tan \chi_e } \right)^2 \cos \left( \theta_e + \chi_e \right) - \cos \left( \theta_e - \chi_e \right) + e \left( b + \frac{e}{2 \tan \chi_e } \right).$$  \hspace{1cm} (29)

The safety against rupture through rubble only considering a straight rupture line can be verified by the following limit state equation (see Fig. 2 (d)):

$$g_c = \text{Minimum } \frac{(\gamma_s - \gamma_w)A_1 + (W_1 - F_s) - (F_h + F_{he})\cot(\phi_r - \theta_s)}{B_z + b + (h_e + e)\cot \chi_e}. $$  \hspace{1cm} (30)

Note that the following constraint is added since the rupture line should be within the rubble mound:

$$0 \leq \theta_s \tan^{-1} \left( \frac{h_n}{B_z + b + (h_n + e)\cot \chi_e} \right).$$  \hspace{1cm} (31)

The safety against rupture through rubble and along top of subsoil (sand) failure can be verified by the following limit state equation (see Fig. 2 (e)), including the leeward armor layer:

$$g_d = (W_1 - F_s)\tan \phi_s + (\gamma_s - \gamma_w)A_2\tan \phi_s - (F_h + F_{he}).$$  \hspace{1cm} (32)

where $\phi_s$ is the reduced effective friction angle of the sand obtained as in Eq. (22), and $A_2$ is the area of the zone 2 shown in Fig. 2 (e), including the leeward armor layer:

$$A_2 = (2(B_z + b + ecot \chi_e) + h_n(cot \chi_e - cot \phi_s))h_n/2 + (b + ecot \chi_e/2)e.$$

Note that in this case the angle between the bottom of the wall and the rupture surface is $\theta_e = \phi_r$.

The safety against rupture through rubble and sand subsoil can be verified using the following limit state equation:

$$g_d = \text{Minimum } \frac{(\gamma_s - \gamma_w)}{2} \omega_{1v} b \omega_{h} + \sum_{i=2}^{4} W_i + (W_1 - F_s)\omega_{1v} - (F_h + F_{he})\omega_{1h},$$  \hspace{1cm} (34)

where $\omega_{1h} = \cos(\phi_r - \theta_s) / \cos(\phi_r)$ and $\omega_{1v} = \sin(\phi_r - \theta_s) / \cos(\phi_r)$ are the horizontal and vertical displacements, respectively, and $W_2, W_3$ and $W_4$ are the work from self weight in zones 2, 3 and 4 (see Fig. 2 (f)), respectively, which can be obtained using the following formula:

$$W_2 = \frac{(\gamma_s - \gamma_w)A_2 \omega_{1v}}{2},$$  \hspace{1cm} (35)

$$W_3 = \frac{(\gamma_s - \gamma_w)}{2 \tan^2 \phi_r + 2} \left[ \frac{r_{DF}^2}{2 \tan^2 \phi_r + 2} \exp(\theta_s \tan \phi_r) \right. \times (\tan \phi_r \sin(\phi_r - \theta_s + \theta_s) - \cos(\phi_r - \theta_s + \theta_s)) - (\tan \phi_r \sin(\phi_r - \theta_s) - \cos(\phi_r - \theta_s))],$$  \hspace{1cm} (36)

$$W_4 = \frac{(\gamma_s - \gamma_w)A_4 \sin(\phi_r + \theta_s - \theta_s)}{\cos \phi_r} \exp(\theta_s \tan \phi_r),$$  \hspace{1cm} (37)

where $A_2$ and $A_4$ are the areas in zones 2 and 4 shown in Fig. 2 (f), respectively, $r_{DF}$ is the length of the line between points D and F, and $\theta_s$ and $\theta_e$ are the angles shown in Fig. 2 (f).

Additionally, the following constraints should be considered:

$$\theta_s \geq \frac{\pi}{2} - \phi_r, \hspace{1cm} (39)$$

where Eq. (38) ensures that the rupture line enters the subsoil.

4.1.3. Overtopping failure

For a composite breakwater of seaboard slope $\chi_s$ and freeboard $h_c$, (see Fig. 2 (g)), and a sea state defined by a significant maximum wave height $H_{\text{max}}$, the mean overtopping volume $q$ per unit of breakwater length is given, for a caisson breakwater, by the exponential relation (see Franco and Franco, 1999)

$$q = a \exp(-(b_{0h}H_{\text{max}})\sqrt{gH_{\text{max}}^3}),$$  \hspace{1cm} (40)

where $q/\sqrt{gH_{\text{max}}^3}$ is the dimensionless discharge, $h_c/H_{\text{max}}$ is the relative freeboard, and $a$ and $b_c$ are coefficients that depend on the structure shape and on the water surface behavior at the seaward face.

The definition of tolerable limits for overtopping is still an open question, given the high irregularity of the phenomenon and the difficulty of measuring it and its consequences. Different levels ranging from functional safety (serviceability limit states) to structural safety (ultimate limit states) mainly in cast in situ concrete superstructures could be considered (see Goda, 1985; Franco et al., 1994).

The safety against overtopping failure can be verified from the following equation:

$$g_o = q_0 - q,$$  \hspace{1cm} (41)

where $q_0$ is the maximum allowable mean overtopping discharge for the considered damage.

4.1.4. Berm instability failure

It is customary in caisson breakwater construction to provide a few rows of foot-protection concrete blocks at the front and rear of the upright section. It usually consists of rectangular blocks weighing from 100 to 400 kN, depending on the design wave height. This protection is indispensable, especially against oblique wave attack. The remainder of the berm and slope of the rubble mound foundation must be protected with armor units of sufficient weight to withstand the wave action. In this paper we take into account only the stability of the berm and the slope, so berm instability failure refers to the removal of pieces from the berm and slope as is shown in Fig. 2 (b).
In Tanimoto et al. (1980) and Tanimoto et al. (1982) the following formula of the Hudson type is proposed:

\[ W_{\text{min}} = \frac{\gamma_s}{N_s^3 (R - 1)^3} H_{\text{max}}^3, \]

(42)

where \( N_s \) is the stability number, \( R \) is a dimensionless constant, which depends on the specific weight of the armor units \( \gamma_s \) (rubble armor units) and on the water unit weight \( \gamma_w \), and \( W \) is the minimum individual armor block weight of the berm, which are given by:

\[ R = \frac{\gamma_s}{\gamma_w}, \]

(43)

\[ N_s \text{max} \left\{ 1.8, 1.3 \left( \frac{1 - c}{e} \right) d^{1/3} H_{\text{max}} + 1.8 \exp \left( -1.5 \frac{d(1 - c)^2}{H_{\text{max}} e^{1/3}} \right) \right\}, \]

(44)

\[ c = \frac{4\pi d}{L \sinh \left( \frac{4\pi d}{L} \right)} c_2, \]

(45)

\[ c_2 = \max \left\{ \sin^2 \theta_w \cos^2 \left( \frac{2\pi x}{L} \cos \theta_w \right), \right\} \]

(46)

\[ \cos^2 \theta_w \sin^2 \left( \frac{2\pi x}{L} \cos \theta_w \right) \right\} \frac{\sin^2 \theta_w \cos^2 \left( \frac{2\pi x}{L} \cos \theta_w \right)}{0 \leq \delta_b}, \]

where \( d \) is the berm depth in front of the caisson, \( c \) and \( c_2 \) are two auxiliary variables, \( \theta_w \) is the wave incidence angle, \( L \) is the wave length at the depth \( d \), \( B_m \) is the seaward berm width, and \( x \) is the distance which gives maximum \( c_2 \). For normal wave incidence (\( \theta_w = 0 \)) \( x \) reduces to \( B_m \). Numerical studies of Eq. (45) suggest that obliquely incident waves may produce more damage to armor units of a rubble mound than waves of normal incidence. Under such conditions, the occurrence of failure can be determined from the following equation:

\[ g_{\text{sa}} = W - \frac{\gamma_s}{N_s^3 (R - 1)^3} H_{\text{max}}^3, \]

(47)

where \( W \) is the real armor unit weight in the seaward berm given by \( \gamma_s J^2 \), where \( J^2 \) is the equivalent cubic block side.

In addition, in order to prevent the seaward failure in calm sea wave conditions, the rupture surface through rubble only is considered. Note that this restraint is considered as a geometric constraint rather than a failure mode in the optimization process. The corresponding verification equation is:

\[ g_{\text{secu}} = \phi_r - \arctan \left( \frac{h_n}{B + B_m + (e + h_n) \cot \theta_n} \right), \]

(48)

where \( B_m \) is the seaward berm width and \( \theta_n \) is the seaward slope angle.

For a detailed description of the derivation of the formulae, see Sorensen and Burcharth (2000) and Oumeraci et al. (2001).

**Remark 4.1.** Note that once the optimal solution has been obtained, the limit state Eqs. (18) (28) (30) (32) (41) (47) and (48) allow one to determine both global and sets of partial safety factors which are equivalent to the 18 reliability constraints in the sense of leading to the same optimal solution (see PIANC, 2003; Burcharth, 2000).

In fact, the proposed method can be extended to include global and partial safety factors. The authors are at present working along these lines, which is the focus of another paper.

4.2. Practical design criteria

In maritime works there are some rules of good practice that should be observed. Some of them are country dependent and some have historical roots, others are taken as a precaution against impulsive breaking wave conditions. Those used in this example, are (see Fig. 1):

1. **Layer slopes and berm widths:** The seaward and leeward berms and slope protection have the following restrictions. The minimum armor unit weight allowed is 0.3 kN while the maximum is 41 kN (concrete pieces have to be used for greater weight armor units), and this implies that the armor layer thickness limits are (\( e = 2l_c \)):

\[ 0.5 \leq e \leq 2.5 \text{ (m)}, \]

(49)

where \( e \) is the equivalent cubic block side for the main layer. The minimum berm widths limits are:

\[ B_m \geq \max (2l_c, 5); \text{ (m)} \quad b \geq 2l_c \text{ (m)}. \]

(50)

Note that the formulas by Tanimoto et al. (1980) and Tanimoto et al. (1982) were developed on the condition that the berm is of reasonable width (such as the 5 m value selected in this paper).

The gradient of the slopes of the rubble mound are usually bounded by:

\[ 1.5 \leq \cot \theta_r \leq 3; \quad 1.5 \leq \cot \theta_r \leq 3. \]

(51)

2. **Construction or operational reasons:** The caisson width limits are:

\[ 10 \leq B \leq 50 \text{ (m)}, \]

(52)

while the maximum and minimum leeward freeboards are, respectively:

\[ 1 \leq h_n + h_s + h_b - h_{lo} - t_r \leq 6 \text{ (m)}, \]

(53)

where \( h_b \) is the concrete crown height, \( h_s \) is the sand-filled caisson height, \( h_n \) is the rubber core height, \( h_{lo} \) is the water depth in front of the caisson corresponding to the zero port reference level (minimum water depth) and \( t_r \) is the tidal range. Note that the lower bound prevents the flooding of the transitable superstructure, whereas the upper bound prevents unrealistic designs form the viewpoint of construction engineers.

The minimum water level in front of the vertical breakwater is given by

\[ h \geq h_{lo}. \]

(54)

In order to facilitate the construction of the caisson crown, the sand-filled caisson crest should exceed the maximum tide level:

\[ h_s + h_e \geq h_{lo} + t_r + 0.1. \]

(55)
The following constraints are used in order for the vertical breakwater to be a composite breakwater:
\[ \frac{h_n + e}{h_0 + t_e} \geq 0.3; \quad \frac{h_n + e}{h_0} \leq 0.9. \] (56)

For structural reasons the ratio between the parapet height \(h_n\) and its width \(w_e\) has to fulfill \(h_n = 1.5w_e\).

(3) Geometric identities:
\[ h = h_1 + h_2; \quad h = h' + h_n; \quad h' + h_{z} = h_0 + h_s + h_o; \]
\[ d + e = h' \] (57)

where \(h\) is the water column in front of the breakwater, \(h_1\) is the water level in front of the breakwater owing to the astronomical tide, and \(h_2\) is the rise of the water level produced by barometrical or storm surge effects (meteorological causes).

4.3. Failure rate upper bounds

It has been shown that the system probabilities of failure during the lifetime of the structures in coastal engineering works (related to geotechnical failure modes) are bigger (for ultimate limit states the range is 0.05 – 0.15 depending on the consequences of failure and the lifetime, see Sorensen and Burcharth, 2000 and ROM, 2001) than other civil engineering structures, such as bridges (probabilities of failure from 10^{-4} to 10^{-7}).

In this paper the selection of the failure rate bounds depends on the consequences of failures; thus, the greater the consequence of failure, the lower the failure rate bound. Note that all are yearly failure rates.

The selected upper failure rate bounds are:

- Operating suspension: \(R^0_0 = 0.005\) (Overtopping);
- Damage: \(R^0_a = 0.003\) (Armor instability);
- Ultimate: \(R^0_s = 0.001; R^0_d = 0.001; R^0_c = 0.001; R^0_\text{ult} = 0.001\) (Geotechnical failure)

where the subindices refer to failure modes.

5. Statistical assumptions

To complete the model, the statistical assumptions need to be provided. They are strongly dependent on the location of the maritime structure. For illustrative purposes, in this section we present those for a composite breakwater in the harbor at Gijón.

5.1. Random and deterministic project factors

The joint distribution of all variables involved is based on the following assumptions (all the numeric values are listed in Table 1):

(1) Optimized design variables: The subset \(\{B, b, h, d, h_n, h_o\}\) of optimized design variables \(d\) related to the concrete caisson are assumed to be deterministic because the construction control is assumed to be good, whereas the subset of variables associated with the rubble mound \(\{b, B_m, e, h_n, x_t, x_o\}\) are considered normal random variables with low uncertainty whose mean values are obtained from the optimization procedure. In what follows the mean value, standard deviation and the coefficient of variation of any variable \(x\) will be denoted as \(\mu_x, \sigma_x\) and \(v_x\), respectively.

(2) Load variables: The joint distribution of the three-variate random variable \((H_{\text{max}}, H_{\text{max}}, T_{\text{max}})\) defining our simplified storms and other factors affecting the incident waves, are defined by definition of (see Appendix C for details):

(a) The marginal cumulative distribution function of \(H_{\text{max}}\). Based on extreme value considerations and the truncated character of the simplified storms (they were considered only for \(H_{\text{max}} \geq 3\)), it is shown in Appendix C that \(H_{\text{max}}\) can be assumed to be a generalized Pareto distribution.

The threshold level 3 m of \(H_{\text{max}}\) is purposely small in order to avoid any influence of this choice on the results. The parameter values of \(k_x\) of the Pareto distribution in Eq. (C.1) is known not to change with the threshold value, but the value of \(\delta_x\) changes with the threshold value. This compensates the large decrease in the number of storms when the threshold value increases.

(b) The conditional distribution \(H_{\text{max}}|H_{\text{max}}\) of \(H_{\text{max}}\) given \(H_{\text{max}}\). Based on a regression analysis combined with a probability paper analysis (see Appendix C), we assume \(H_{\text{max}}|H_{\text{max}}\) to be a maximal Weibull distribution.

(c) The conditional distribution \(T_{\text{max}}|H_{\text{max}}, H_{\text{max}}\) of \(T_{\text{max}}\) given \(H_{\text{max}}|H_{\text{max}}\). Based on a regression analysis combined with a probability paper analysis (see Appendix C), the distribution of \(T_{\text{max}}|H_{\text{max}}, H_{\text{max}}\) is assumed to be normal.

(d) The water depth in front of the breakwater \(h_1\), considering the tidal elevation, is modelled as (see (3.2) a mixture of two random normal variables \(N(\mu_{h_1}, \sigma_{h_1})\) and \(N(\mu_{h_1}, \sigma_{h_1})\) with weights \(\alpha\) and \(1 - \alpha\), respectively, and truncated on the left at \(h_0\) m. and on the right at \(h_0 + t_e\) m, where \(h_0\) is the minimum value of \(h_1\) (water depth in front of the breakwater corresponding to the zero port reference level) and \(t_e\) is the tidal range, and the parameters have been estimated using data from the tidal gauge at Gijón. The cumulative distribution function used in the Rosenblatt transformation is:

\[ F(h_1) = \frac{\phi \left( \frac{h_1 - h_0 - \mu_{h_1}}{\sigma_{h_1}} \right) - \phi \left( \frac{- \mu_{h_1}}{\sigma_{h_1}} \right)}{\phi \left( \frac{t_e - \mu_{h_1}}{\sigma_{h_1}} \right) - \phi \left( \frac{- \mu_{h_1}}{\sigma_{h_1}} \right)} + (1 - \alpha) \frac{\phi \left( \frac{h_1 - h_0 - \mu_{h_1}}{\sigma_{h_1}} \right) - \phi \left( \frac{- \mu_{h_1}}{\sigma_{h_1}} \right)}{\phi \left( \frac{t_e - \mu_{h_1}}{\sigma_{h_1}} \right) - \phi \left( \frac{- \mu_{h_1}}{\sigma_{h_1}} \right)} \]

(e) The water level rise owing to meteorological causes \(h_2\) is assumed to be a normal random variable with mean \(\mu_{h_2}\) and standard deviation \(\sigma_{h_2}\).

(f) The incident wave angle \(\theta_w\) is assumed to be normal \(N(0, \sigma_{\theta_w}^2)\).
(g) The coefficient $A$ in Eq. (16) for modelling the change in the maximum wave height due to random wave breaking is modelled as a normal random variable. As there is no clear variation are not taken into account.

(c) The average unit weight of the sand-filled caisson $\gamma_c$ and the unit weight of the rubble $\gamma_s$ are considered normal random variables with means $\mu_{\gamma_c}$, $\mu_{\gamma_s}$ and standard deviations $\sigma_{\gamma_c}$, $\sigma_{\gamma_s}$, respectively. The spatial variation is not taken into account.

(3) The soil strength is modelled using the following assumptions:

(a) The friction factor $\mu_\ell$ between the caisson base and the rubble is assumed log-normal distributed with mean $\mu_\ell$ and standard deviation $\sigma_{\mu_\ell}$.

(b) Since the breakwater foundation is made of friction material, a statistical model for the angle of internal friction of rubble and sand is required. These angles are modelled by log-normal random variables considering the effective friction and dilation angles with means $\mu_\ell$, $\mu_{\ell_2}$, $\mu_g$, $\mu_{\mu_g}$ and $\mu_{\sigma_g}$, respectively. The coefficients of variation are $\mu_{\ell_2}$, $\mu_{\ell_2_2}$, $\mu_g$, $\mu_{\mu_g}$ and $\mu_{\sigma_g}$, respectively. The spatial variation is not taken into account.

(4) In an attempt to consider all the sources of uncertainty, the uncertainties of the formulas used in the computations have to be examined (see Oumeraci et al., 2001). Some models are based on empirical relations and show a certain scatter, whereas others are physically based on assumptions or simplifications.
In any case, a calibration factor is applied to the result of the formula providing the true value:

(a) The Goda formulae for pulsating wave forces are biased in order to provide a safe relation (see Van Der Meer et al., 1994; Oumeraci et al., 2001). The uncertainty is taken into account using the calibration factors \( A_g, B_g, M_{Ag}, M_{Bg} \) and \( S_g \) affecting horizontal forces \( (F_h) \), uplift forces \( (F_v) \), horizontal moments \( (M_h) \), uplift moments \( (M_v) \) and seepage horizontal forces, respectively.

(b) The reliability of the overtopping prediction formula (40) can be expressed assuming a normal distribution for the random variable \( b_o \), thus \( b_o \sim N(\mu_{b_o}, \sigma_{b_o}) \) (see Franco and Franco, 1999). Note that the coefficient \( a \) in Eq. (40) is considered deterministic.

(c) The uncertainty in the Hudson type formulae (42) is considered to be due to the normal random coefficient \( C_{ar} \sim N(\mu_{C_{ar}}, \sigma_{C_{ar}}^2) \).

(5) To consider model uncertainties for the limit state equations, model factors equivalent to global safety factors are considered. These will be random parameters \( F_m \) (\( m \) refers to failure mode) log-normally distributed with expected values \( \mu_{F_m} \) and coefficients of variation \( \nu_{F_m} \) (see Oumeraci et al., 2001). Note, for example, that in the overtopping failure, \( F_o \) takes into account the uncertainty of the critical structural safety discharge \( q_0 \).

All these assumptions and the numeric values used in the example are listed in Table 1.

5.2. Dependence assumptions

The group of random variables \( \{H_{smax}, H_{max}, T_{zmax}\} \) are assumed to be dependent with the marginal and conditional distributions given above. For the sake of simplicity, the tidal water level is assumed to be independent of the remaining variables, and the same assumption is used for the meteorological tide; note, however, that this hypothesis is a simplification because in fact it is dependent on \( H_{smax} \). The same would be applicable if storm surge effect in shallow waters were considered.

The remaining variables will be considered independent in this paper though, for example, some authors (see Burcharth and Sorensen, 1998) consider there to be dependence between \( F_h \) and \( M_h \) and \( F_v \) and \( M_v \). It is important to remember here that, in addition, the correlation of the different modes of failure stems from the fact that they depend on common variables that can be dependent or independent. Thus, even in the case of assuming independent variables, the modes of failure will become correlated because of their dependence on common variables. In other words, the main source of the different modes of failure correlations is their dependence on common variables and not the dependence on the variables themselves.

The above probability functions and the value of their parameters have been chosen solely for illustration purposes.

Table 2

<table>
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<tr>
<th>( i )</th>
<th>( X_i )</th>
<th>Meaning</th>
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<td>( C_{wo} )</td>
<td>Sand filled caisson construction cost per unit volume</td>
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<td>5</td>
<td>( C_{co} )</td>
<td>Rubble core construction cost per unit volume</td>
<td>10</td>
<td>$/m^3</td>
</tr>
<tr>
<td>6</td>
<td>( q_{0} )</td>
<td>Maximum allowable mean overtopping discharge for structural damage</td>
<td>0.2</td>
<td>m³/s/m.l.</td>
</tr>
<tr>
<td>7</td>
<td>( R_{o} )</td>
<td>Overtopping repair rate, ( m = o )</td>
<td>0.005</td>
<td>year⁻¹</td>
</tr>
<tr>
<td>8</td>
<td>( R_{a} )</td>
<td>Armor failure rate upper bound, ( m = a )</td>
<td>0.003</td>
<td>year⁻¹</td>
</tr>
<tr>
<td>9</td>
<td>( r_{st} )</td>
<td>Mean number of storms per year</td>
<td>45.3427</td>
<td>storms/year</td>
</tr>
<tr>
<td>10</td>
<td>( y_{sp} )</td>
<td>Superstructure concrete unit weight</td>
<td>23</td>
<td>kN/m³</td>
</tr>
<tr>
<td>11</td>
<td>( y_{w} )</td>
<td>Water unit weight</td>
<td>10.35</td>
<td>kN/m³</td>
</tr>
<tr>
<td>12</td>
<td>( \tan \theta_h )</td>
<td>Mean angle tangent of the sea bottom</td>
<td>1/50</td>
<td>– –</td>
</tr>
</tbody>
</table>
In order to apply the method to real cases, a more careful selection has to be done, using long term data records. Only a few countries have enough information to infer these functions adequately.

6. Numerical example

The proposed method has been implemented in GAMS (General Algebraic Modelling System) (see Castillo et al., 2001). GAMS is a software system specially designed for solving optimization problems (linear, non-linear, integer and mixed integer) of small to very large size. All the examples have been solved using the generalized reduced gradient method (for more details see Vanderplaats, 1984 or Bazaraa et al., 1990) that has shown good convergence properties including constraints to the variables. The main advantages of GAMS are:

1. It is a high quality software package (reliable, efficient, fast, widely tested, etc.).

![Diagram](image-url)

Fig. 4. Resulting optimal solution for the vertical breakwater example.
Unlike FORM level II methods the proposed method does not need to invert the Rosenblatt transformation and the failure region need not be written in terms of the normalized transformed variables.

Of course, other optimization programs such as AIMMS (Bischop and Roelofs, 1999), AMPL (Fourer et al., 1993), LINDO, What’s Best, MPL or the Matlab Optimization Toolbox, can be used instead.

To illustrate the method, the automatic optimal design (see Fig. 1) of a composite breakwater with the statistical model and random model parameters $\kappa$ and the fixed deterministic parameters shown in Tables 1 and 2, respectively, has been performed. Note that the maximum yearly failure rates have been defined depending on the importance of the corresponding failure.

From the analysis of results, the following conclusions can be drawn from the analysis of the results:

(1) The proposed method leads to the solution of the breakwater design problem showing a good behavior. Note that the number of reliability evaluations is 63 (7 failure modes $\times$ 9 iterations), lower than the typical...
number used in these kind of problems (100–500) (see Voortman et al., 1998). Note also that in the computational example 10 design variables, 35 statistical variables and 7 failure modes have been considered.

(2) Table 3 shows the convergence of the process that is attained after 9 iterations with an error tolerance lower than $1 \times 10^{-7}$. The first column shows the initial construction cost ($C_0$), the optimized design variables $d$, and the yearly failure rates of the different failure modes resulting after each iteration. The last column gives the corresponding final values. The resulting optimal breakwater design is shown in Fig. 4.

<table>
<thead>
<tr>
<th>$d_i$</th>
<th>$\frac{\partial r_{fa}}{\partial d_i}$</th>
<th>$\frac{\partial r_{fa}}{\partial d_i}$</th>
<th>$\frac{\partial r_{fa}}{\partial d_i}$</th>
<th>$\frac{\partial r_{fa}}{\partial d_i}$</th>
<th>$\frac{\partial r_{fa}}{\partial d_i}$</th>
<th>$\frac{\partial r_{fa}}{\partial d_i}$</th>
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</thead>
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<td>$b$</td>
<td>–</td>
<td>–</td>
<td>$-0.000192$</td>
<td>$-0.000107$</td>
<td>$-0.001284$</td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td>$B$</td>
<td>$-0.004348$</td>
<td>$-0.008655$</td>
<td>$-0.001718$</td>
<td>$-0.002127$</td>
<td>$-0.005488$</td>
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<td>–</td>
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<tr>
<td>$d_a$</td>
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<td>–</td>
<td>–</td>
<td>–</td>
<td>$0.018256$</td>
<td>–</td>
</tr>
<tr>
<td>$e$</td>
<td>$0.000365$</td>
<td>$0.000472$</td>
<td>$-0.000115$</td>
<td>$0.000064$</td>
<td>$-0.000965$</td>
<td>–</td>
<td>$0.022008$</td>
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<tr>
<td>$h_b$</td>
<td>$-0.001119$</td>
<td>$-0.00713$</td>
<td>$-0.000182$</td>
<td>$-0.000408$</td>
<td>$0.000279$</td>
<td>$-0.012560$</td>
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<tr>
<td>$h_a$</td>
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<td>$-0.002465$</td>
<td>$-0.017730$</td>
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<td>$h_s$</td>
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<td>$0.000483$</td>
<td>$0.000105$</td>
<td>$0.000176$</td>
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<td>$s_i$</td>
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<td>$0.000840$</td>
<td>$0.000245$</td>
<td>$0.004256$</td>
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<tr>
<td>$s_s$</td>
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Table 7

<table>
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<tr>
<th>$\bar{\eta}_1$</th>
<th>$\frac{\partial r_{fa}}{\partial \bar{\eta}_1}$</th>
<th>$\frac{\partial r_{fa}}{\partial \bar{\eta}_1}$</th>
<th>$\frac{\partial r_{fa}}{\partial \bar{\eta}_1}$</th>
<th>$\frac{\partial r_{fa}}{\partial \bar{\eta}_1}$</th>
<th>$\frac{\partial r_{fa}}{\partial \bar{\eta}_1}$</th>
<th>$\frac{\partial r_{fa}}{\partial \bar{\eta}_1}$</th>
<th>$\frac{\partial r_{fa}}{\partial \bar{\eta}_1}$</th>
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<tr>
<td>$\mu_A$</td>
<td>$0.002656$</td>
<td>$0.004258$</td>
<td>$0.001020$</td>
<td>$0.002104$</td>
<td>$0.003677$</td>
<td>–</td>
<td>$0.020948$</td>
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<td>$\mu_I$</td>
<td>$0.003615$</td>
<td>$0.003529$</td>
<td>$0.000324$</td>
<td>$0.001742$</td>
<td>$0.003741$</td>
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<tr>
<td>$\mu_{a}$</td>
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<td>–</td>
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<td>–</td>
<td>–</td>
<td>$-0.028891$</td>
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<td>$\mu_{b}$</td>
<td>$0.000751$</td>
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<td>$0.000298$</td>
<td>$-0.000490$</td>
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<td>–</td>
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<td>$\mu_{c}$</td>
<td>–</td>
<td>$0.004463$</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>$0.008555$</td>
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</tr>
<tr>
<td>$\mu_{d}$</td>
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<td>$0.001631$</td>
<td>–</td>
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<td>$\mu_{e}$</td>
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<td>–</td>
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<td>–</td>
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<td>–</td>
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<td>–</td>
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<tr>
<td>$\mu_{g}$</td>
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<td>$\mu_{h}$</td>
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<td>–</td>
<td>–</td>
<td>–</td>
<td>$0.007446$</td>
<td>–</td>
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<tr>
<td>$\mu_{i}$</td>
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<td>$0.000003$</td>
<td>$0.000001$</td>
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<td>$0.000051$</td>
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<td>$0.001149$</td>
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<td>$\mu_{A_{nd}}$</td>
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<td>$\mu_{s}$</td>
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<td>–</td>
<td>–</td>
<td>–</td>
<td>–</td>
</tr>
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<td>$\mu_{v}$</td>
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<td>–</td>
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<td>$\mu_{w}$</td>
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<td>$\mu_{y}$</td>
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<td>$a$</td>
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<td>$0.007445$</td>
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<tr>
<td>$C_{a_{id}}$</td>
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<td>–</td>
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<tr>
<td>$C_{a_{id}}$</td>
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<td>$C_{a_{id}}$</td>
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<td>–</td>
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<td>$0.000146$</td>
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<td>$0.004386$</td>
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<td>$0.051990$</td>
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<td>$\gamma_{sp}$</td>
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<td>$-0.001784$</td>
<td>$-0.000412$</td>
<td>$-0.000728$</td>
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<td>$0.000271$</td>
<td>$0.000492$</td>
<td>–</td>
<td>$0.002547$</td>
</tr>
</tbody>
</table>
(3) It is interesting to see that in the first iterations the failure rate constraints do not hold (they are greater than the failure rate bounds \( R_m^0 \), the construction cost is increased in order to increase the safety levels until the final design is obtained, where all the reliability constraints hold.

(4) At the optimal solution four reliability constraints are active: rotation failure \((b)\), rupture surface through rubble and sand subsoil \((rs)\), overtopping \((o)\) and armor instability failure \((a)\). Considering the modes associated with geotechnical failures (ultimate limit states), the bounds given by the formula in Eqs. (8) and (9), the

\[
\begin{align*}
\frac{\partial r_i}{\partial k_i} \left| \begin{array}{c}
\text{Table 8} \\
\end{array} \right| \\
\end{align*}
\]

Sensitivities of the yearly failure rates \( r_m \) with respect to the parameters

\[
\begin{align*}
\begin{array}{cccccccc}
\kappa_i & \frac{\partial r_i}{\partial k_i} \left| \begin{array}{c}
\text{($\%$)} \\
\end{array} \right| & \frac{\partial r_i}{\partial k_i} \left| \begin{array}{c}
\text{($\%$)} \\
\end{array} \right| & \frac{\partial r_i}{\partial k_i} \left| \begin{array}{c}
\text{($\%$)} \\
\end{array} \right| & \frac{\partial r_i}{\partial k_i} \left| \begin{array}{c}
\text{($\%$)} \\
\end{array} \right| & \frac{\partial r_i}{\partial k_i} \left| \begin{array}{c}
\text{($\%$)} \\
\end{array} \right| & \frac{\partial r_i}{\partial k_i} \left| \begin{array}{c}
\text{($\%$)} \\
\end{array} \right| & \frac{\partial r_i}{\partial k_i} \left| \begin{array}{c}
\text{($\%$)} \\
\end{array} \right| & \frac{\partial r_i}{\partial k_i} \left| \begin{array}{c}
\text{($\%$)} \\
\end{array} \right| \\
\end{array}
\end{align*}
\]

- \( v_b \) 0.000010
- \( v_{bn} \) 0.000015
- \( v_c \) 0.000006 0.000007
- \( v_s \) 0.000002
- \( k_r \) 0.000004 0.000016
- \( \delta_s \) 0.000002
- \( \lambda_s \) 0.000021
- \( \alpha \) 0.000008 0.000017
- \( \beta_r \) 0.000007 0.000042
- \( \sigma_r \) 0.000002
- \( \sigma_a \) 0.000036
- \( \sigma_o \) 0.000012
- \( \sigma_i \) 0.000002
- \( \sigma_s \) 0.000056
- \( \sigma_f \) 0.000009
- \( \sigma_d \) 0.000017
- \( \sigma_r \) 0.000032
- \( \sigma_o \) 0.000004
- \( \sigma_s \) 0.000032
- \( \sigma_d \) 0.000037
- \( \sigma_f \) 0.000016
- \( \sigma_r \) 0.000056
- \( \sigma_d \) 0.000009
- \( \sigma_f \) 0.000034
- \( \sigma_o \) 0.000017
- \( \sigma_s \) 0.000016
- \( \sigma_d \) 0.000001
- \( \sigma_f \) 0.000003
- \( \sigma_o \) 0.000042
- \( \sigma_s \) 0.000001
- \( \sigma_d \) 0.000002
- \( \sigma_f \) 0.000004
- \( \sigma_o \) 0.000002
- \( \sigma_s \) 0.000021
- \( \sigma_d \) 0.000007
- \( \sigma_f \) 0.000022
- \( \sigma_o \) 0.000002
- \( \sigma_s \) 0.000010
- \( \sigma_d \) 0.000023
- \( \sigma_f \) 0.000003
- \( \sigma_o \) 0.000011
- \( \sigma_s \) 0.000017
- \( \sigma_d \) 0.000035
- \( \sigma_f \) 0.000017
- \( \sigma_o \) 0.000480
- \( \sigma_s \) 0.000062
- \( \sigma_d \) 0.000037
- \( \sigma_f \) 0.000029
- \( \sigma_o \) 0.000153
system probability of failure during a lifetime of 50 years is bounded between 5% and 15.5%, which is between the reasonable bounds for this type of structures (see Sorensen and Burchart, 2000, and ROM, 2001).

(5) Table 4 shows the maximum likelihood ($z_0$) and the failure points per failure mode, that is the most probable values of the random variables during a storm, and the most probable values of the random variables that induce the failure of the structure, also known as design points (solutions of problems (5) and (6)). The information provided in this table is extremely useful. See, for example, that the most probable significant wave height during a storm $H_{s_{\text{max}}}$ is 3.32 (m), but the wave conditions which cause the failure of the structure are characterized by design wave conditions with significant and maximum wave heights between 10.06 and 11.23 (m), and 18.73 and 20.92 (m), respectively, depending on the failure mode considered, which lead to partial safety factors $^{2}$ in the ranges from 3.03 to 3.38, and 3.27 to 3.65, respectively. These values are slightly bigger than the design wave conditions which caused disasters in vertical breakwaters built before World War II (see Oumeraci, 1994). Note that the ratios between $H_{\text{max}}$ and $H_{s_{\text{max}}}$ range from 1.861 to 1.865, which confirms the practical rule of using a factor between 1.8 and 1.9.

(6) The optimal breakwater shown in Fig. 4 is designed for a very rough sea, its dimensions are slightly bigger than the widest caisson breakwater at Hedono Port (Japan), whose caisson is 38 (m) wide based on the following wave characteristics: 9.7 (m) of significant wave height, 17 (m) of maximum wave height, and 13.2 (s) of significant wave period.

(7) The cost sensitivities with respect to the cost of materials and some parameters of the model ($\eta$ and $\kappa$) are given in Table 5. They allow one to know how much a small change in a single design factor value changes the optimal expected cost per running meter of the composite breakwater. This information is extremely useful during the construction process to control the cost, and for analyzing how the changes in the yearly failure rates required by the codes influence the total cost of maritime works. For example, a change of one unit in the cost of the concrete superstructure $C_{\text{sp}}$ leads to a relative cost increase of $16,174.3$ (see the corresponding entry in Table 5). Similarly, while an increase in the unit weight of the rubble mound $\mu_{\text{sp}}$ decreases the cost ($-147,452.8$), water depth corresponding to the zero port ($h_{\text{lo}}$) increases the cost by $139,534.4$ per relative unit increase. Note that the most restrictive failure mode is that of rotation ($b$) because its absolute value sensitivity is the greatest ($31,828.3$), where obviously, if the maximum yearly failure rate is increased, the cost decreases (negative value of the sensitivity). As the weakest link in the optimal design is the subsoil failure through the rubble (rotation failure $b$), the relative sensitivity with respect to the parameters related to the rubble effective friction angle parameters ($\mu_{\text{sp}}, \nu_{\text{sp}}$) are the greatest in absolute value $-249,507.4$ and $-20,780.9$; thus, it can be concluded that the uncertainty in the subsoil properties plays a very important role (see Voortman et al., 1999).

(8) The sensitivities of the yearly failure rates with respect to the optimized design variables and $d$, non-optimized design variables $\eta$ and parameters $\kappa$ are given in Tables 6, 7 and 8, respectively. As an example, the influence of the freeboard on the verification equation for overtopping (41) and, therefore, on the corresponding yearly failure rate will be analyzed. This equation shows how this failure occurrence depends only on $h_c$, $q_0$, $a$, $b$, and $H_{\text{max}}$. Note that increasing only the freeboard will lead to a safer structure. The freeboard is defined as $h_c = h_0 + h_b + h_z + h_o - h$ with $h = h_1 + h_2$ and $h_{\text{lo}} \leq h$. An increase of any of the variables related to water depth $h_{\text{lo}}$, $t_o$, $h_1$, $h_2$ will produce an increase of the yearly failure rate for overtopping while any increase of any of the variables related to breakwater heights $h_w$, $h_p$, $h_s$ generates a decrease (see the corresponding entries in Table 6) of the yearly failure rate for overtopping due to the fact that all of them appear in the freeboard definition with negative sign for water depths and positive sign for breakwater heights, respectively.

7. Conclusions

The methodology presented in this paper provides a rational and systematic procedure for automatic and optimal design of maritime works. The engineer is capable of observing simultaneous bounds for the yearly failure rates, so that the most stringent conditions prevail. In addition, a sensitivity analysis can be easily performed by transforming the input parameters into artificial variables, which are constrained to take their associated constant values. The provided example illustrates how this procedure can be applied and proves that it is very practical and useful.

Some additional advantages of the proposed method are:

(1) The method allows an easy connection with optimization frameworks.

(2) The responsibility for iterative methods is given to the optimization software.

(3) The reliability analysis takes full advantage of the systematic procedure for automatic and optimal design of linear, non-linear, mixed-integer problems. The designer simply needs to choose the adequate optimization algorithm.

In this paper the partial safety factor is the ratio between the failure point and the most probable value of the random variable considered ($z_0$).

2 In this paper the partial safety factor is the ratio between the failure point and the most probable value of the random variable considered ($z_0$).
Acknowledgments

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Appendix A. Set of variables

As an illustration, for the composite breakwater example (see Fig. 1), the optimized design variables, \( d \), are the geometric variables defining the dimensions of the main elements of the composite breakwater, i.e., (see Fig. 1):

\[
d = \{b, B, B_m, e, h_b, h_s, h_o, z_l, z_s\},
\]

(A.1)

the non-optimized design variables, \( \eta \), include other geometric variables, costs of materials, unit weights, etc., i.e.:

\[
\eta = \{a, A, A_g, b_0, B_g, C_d, C_{gw}, C_{ca}, C_{co}, C_{sp}, F_m, h_1, h_2, H_{max}, H_{sum}\}
\]

\[
\bigcup \{M_{a}, B_{a}, F_{0}, r_{st}, R_{w}, S_{g}, T_{max}, \omega_{o}, \gamma_{o}, \gamma_{sp}, \gamma_{w}, \theta_{w}, \theta_{m}\}
\]

\[
\bigcup \{\mu_{c}, \phi_{r}, \phi_{s}, \varphi_{r}, \varphi_{s}, \psi_{r}, \psi_{s}\},
\]

the random model parameters, \( \kappa \), include the coefficients of variations, standard deviations, parameters of the joint probability density function, etc.: 

\[
\kappa = \{v_{o}, v_{ho}, \varphi_{o}, \varphi_{ho}, \varphi_{h}, \kappa_{r}, \delta_{r}, \delta_{w}, \delta_{w}, \alpha_{r}, \alpha_{w}, \alpha_{l}, \gamma_{w}, \gamma_{l}, \gamma_{o}, \gamma_{sp}, \gamma_{w}, \theta_{w}, \theta_{m}\}
\]

\[
\bigcup \{\sigma_{T_{max}}, \sigma_{h_0}, \sigma_{\mu_{c}}, \sigma_{\phi_{r}}, \sigma_{\phi_{s}}, \sigma_{\varphi_{r}}, \sigma_{\varphi_{s}}, \sigma_{\psi_{r}}, \sigma_{\psi_{s}}\}
\]

\[
\bigcup \{v_{\gamma_{o}}, v_{\gamma_{ho}}, v_{\gamma_{w}}, v_{\gamma_{sp}}, v_{\gamma_{w}}, v_{\theta_{w}}, v_{\theta_{m}}, \sigma_{\gamma_{o}}, \sigma_{\gamma_{ho}}, \sigma_{\gamma_{w}}, \sigma_{\gamma_{sp}}, \sigma_{\gamma_{w}}, \sigma_{\theta_{w}}, \sigma_{\theta_{m}}\}
\]

\[
\bigcup \{\sigma_{\gamma_{o}}, \sigma_{\gamma_{ho}}, \sigma_{\gamma_{w}}, \sigma_{\gamma_{sp}}, \sigma_{\gamma_{w}}, \sigma_{\theta_{w}}, \sigma_{\theta_{m}}\}
\]

and the dependent variables \( \psi \), include redundant geometric variables, volumes, moments, etc., which can be written in terms of variables \( d \) and \( \eta \):

\[
\psi = \{A_1, A_2, A_3, A_4, B_c, B_w, c, c_2, c_4, x, d, F_b, F_c, F_{but}, h, h_0, \rho_{hc}, H_{break}, H_3\}
\]

\[
\bigcup \{l_0, L_0, M_0, P_0, N_0, P_3, p_3, p_4, p_5, q, r_{st}, R_{w}, S_{g}, T_{max}, V_{ca}, V_{co}, V_{sp}, V_{al}, V_{co}\}
\]

\[
\bigcup \{W, W_1, W_2, W_3, W_4, x_1, y_{F_{ho}}, y_{F_{o}}, \theta_0, \theta_1, \theta_2, \theta_3, \theta_4, \theta_5, \psi_{r}, \psi_{s}\}
\]

Appendix B. Cost function

Consider the composite breakwater in Fig. 1. To derive the cost function the following parts are considered:

Concrete volume: The caisson and superstructure volumes are:

\[
V_{ca} = Bh_s; V_{sp} = Bh_b + \rho_{ho}h_o.
\]

(B.1)

Armor layer volume: The armor layer volume is:

\[
V_{al} = e[B_m + b + h_n(\tan\theta_n + 1/\tan\theta_l)] + 0.5e(1/\tan\theta_n + 1/\tan\theta_l).
\]

(B.2)

Core volume: The core volume is:

\[
V_{co} = h_n(b + B + B_m - e(\tan\theta_n/2 + \tan\theta_l/2)) + h_n(1/\tan\theta_n + 1/\tan\theta_l)/2).
\]

(B.3)

Then, the construction cost per unit length becomes:

\[
C_0 = C_{ca}V_{ca} + C_{sp}V_{sp} + C_{al}V_{al} + C_{co}V_{co}.
\]

(B.4)

Appendix C. Statistical definition of storms

In this appendix we derive the joint distribution of \( H_{\text{sum}}, H_{\text{max}}, T_{z\text{max}} \) based on Gijon buoy data. The data correspond to 5.69 years of observations.

The analysis was done as follows:

(1) First, data record with one missed data point were completed by interpolation. This led to the recovery of 6 data records and discarded only three incomplete and not extreme storms.

(2) A threshold level of \( H_s = 3 \) m was selected and a storm defined for the duration of wave height conditions above this level without going down. This led to 258 storms (with significant wave height > 3 m) which implies a yearly rate \( r_w = 45.3427 \) storms/year.

(3) The peak value \( H_{\text{max}} \) of \( H_s \) and the maximum pair observed wave height \( H_{\text{max}} \) and associated zero-up-crossing mean period \( T_{z\text{max}} \) during each storm were registered. This means that a sample of three values \( H_{\text{sum}}, H_{\text{max}} \) and \( T_{z\text{max}} \) per storm was used.

From these data, the joint distribution of the three-variate random variable \( (H_{\text{sum}}, H_{\text{max}}, T_{z\text{max}}) \) need to be defined. Instead of defining the joint density or cumulative distribution function of \( (H_{\text{sum}}, H_{\text{max}}, T_{z\text{max}}) \), which is difficult to visualize, without loss of generality, we define:

(1) The marginal distribution of \( H_{\text{sum}} \).

(2) The conditional distribution \( H_{\text{max}} \mid H_{\text{sum}} \) of \( H_{\text{max}} \) given \( H_{\text{sum}} \).

(3) The conditional distribution \( T_{z\text{max}} \mid H_{\text{max}} \), \( H_{\text{sum}} \) of \( T_{z\text{max}} \) given \( H_{\text{max}}, H_{\text{sum}} \).

The selection of the adequate marginal and conditional distributions is based on extreme value theory and probability paper techniques (see Castillo et al., 2005; Castillo and Sarabia, 1992), as shown below.

C.1. Marginal distribution of \( H_{\text{sum}} \)

To make a proper selection of the marginal distribution of \( H_{\text{sum}} \) and since the data come from left truncation at 3 m of a
sample of maxima, first we use theoretical considerations that lead to a maximal generalized Pareto distribution (GPD) with cumulative distribution function:

\[
F_{H_{\text{max}}}(H_{\text{max}}) = 1 - \left(1 - \frac{\kappa_s(H_{\text{max}} - \lambda_s)}{\delta_s}\right)^{1/\kappa_s};
\]

\[
1 - \frac{\kappa_s(H_{\text{max}} - \lambda_s)}{\delta_s} \geq 0. \tag{C.1}
\]

Thus, we have fitted a GPD by least squares and obtained the following estimates:

\[
\hat{\kappa}_s = -0.1197; \quad \hat{\delta}_s = 0.446; \quad \hat{\lambda}_s = 3.
\]

To check the goodness of the model we have plotted the data on P–P and Q–Q plots, as shown in Fig. C.1. The plots show a reasonable fit, so that based on theory and data evidence we accept the model (C.1). The P–P and Q–Q plot are used for showing the data goodness of fit to a model. This can be done with probability papers when they exist, but this is not the case for the Pareto family. So, we have used these plots instead.

C.2. Conditional distribution of \(H_{\text{max}}\) given \(H_{\text{smax}}\)

To choose a conditional distribution \(H_{\text{max}}|H_{\text{smax}}\) of \(H_{\text{max}}\) given \(H_{\text{smax}}\), we first plot the data \((H_{\text{smax}}, H_{\text{max}})\) and observe that they exhibit a linear regression (see Fig. C.2):

\[
H_{\text{max}} = a_t + b_t H_{\text{smax}}, \tag{C.2}
\]

where the estimated parameters are \(\hat{a}_t = -0.641855\) and \(\hat{b}_t = 1.92856\).

Next, we calculate the residuals and find that they follow a maximal Weibull model (see the Maximal Weibull probability plot in Fig. C.3):

\[
F_X(x) = \exp\left[-1 - \kappa_w\left(\frac{x - \lambda_w}{\delta_w}\right)^{1/\kappa_w}\right];
\]

\[
1 - \kappa_w\left(\frac{x - \lambda_w}{\delta_w}\right) \geq 0. \tag{C.3}
\]

Combining this expression with the regression Eq. (C.2) leads to the final model for \(H_{\text{max}}|H_{\text{max}}\):

\[
F_{H_{\text{max}}|H_{\text{max}}}(x|y) = \exp\left[-1 - \kappa_w\left(\frac{x - a_t - b_t y - \lambda_w}{\delta_w}\right)^{1/\kappa_w}\right]. \tag{C.4}
\]

only valid for

\[
1 - \kappa_w\left(\frac{x - a_t - b_t y - \lambda_w}{\delta_w}\right) \geq 0.
\]

Then, the estimation of the Weibull parameters using the maximum likelihood method leads to

\[
\hat{\kappa}_w = 0.175482; \quad \hat{\delta}_w = 0.470151; \quad \hat{\lambda}_w = -0.201646.
\]

C.3. Conditional distribution of \(T_{z_{\text{max}}}\) given \(H_{\text{max}},\) and \(H_{\text{smax}}\)

To derive the conditional distribution \(T_{z_{\text{max}}}|H_{\text{max}}, H_{\text{smax}}\) of \(T_{z_{\text{max}}}\) given \(H_{\text{max}}, H_{\text{smax}}\), we first tried several regression models for \(T_{z_{\text{max}}}\) given \(H_{\text{max}}, H_{\text{smax}}\) and find as the best model

\[
T_{z_{\text{max}}} = a_t + b_t H_{\text{smax}} + c_t H_{\text{max}}. \tag{C.5}
\]

where the estimated parameters are \(\hat{a}_t = 5.66953\) and \(\hat{b}_t = 3.5765\) and \(\hat{c}_t = -1.35536\).

Next, we obtain the residuals \(e_i; i = 1, \ldots, 258\) (see Fig. C.4) and plot them on a normal probability paper, obtaining the plot in Fig. C.5, which confirms their normality. Once we have
estimated the corresponding parameters by maximum likelihood, we get
\[ e_i \sim N\left(0, \sigma^2_{\tau_{\text{max}}} \right), \]
where \( \sigma_{\tau_{\text{max}}} = 1.6128 \) that leads to the final model for
\[ \tau_{\text{max}} \mid H_{\text{max}}, H_{\text{smax}} \sim N\left(a_1 + b_1 H_{\text{smax}} + c_1 H_{\text{max}}, \sigma^2_{\tau_{\text{max}}} \right). \]  

**Appendix D. Simulation of random storms**

For the sake of completeness and clarifying the definition of the joint distribution of \( H_{\text{smax}}, H_{\text{max}} \) and \( T_{\tau_{\text{max}}} \), we include here how the simulation of these three random variables can be done.

If one is interested in simulating random storms, i.e., random values of \( (H_{\text{smax}}, H_{\text{max}}, T_{\tau_{\text{max}}}) \), for example to run a Monte Carlo simulation, one can use the following algorithmic process:

1. Simulate \( H_{\text{smax}} \) using
\[ H_{\text{smax}} = k_S + u_1 \sigma_S \left(1 - (1 - u_1)^{\kappa_S}\right), \]
where \( u_1 \) is a random uniform \( U(0, 1) \) number.

2. Simulate \( H_{\text{max}} \) using
\[ H_{\text{max}} = a_r + b_r H_{\text{smax}} + \frac{\delta_w}{K_w} (1 - (1 - \log(u_2))^\kappa_w), \]
where \( u_2 \) is a random uniform \( U(0, 1) \) number independent of \( u_1 \).

3. Simulate \( T_{\tau_{\text{max}}} \) using
\[ T_{\tau_{\text{max}}} = a_t + b_t H_{\text{smax}} + c_t H_{\text{max}} + \nu, \]
where \( \nu \) is a random normal \( N(0, 1.6128^2) \) number.

The process must be repeated as many times as the number of desired storms (sample size).

**Appendix E. Solving the optimization problem using decomposition techniques**

In this appendix we give a detailed explanation of how the optimization problem (12)–(15) can be solved.

The problem described in Eqs. (12)–(15) presents some difficulties because constraints (13) require the knowledge of \( r_{\text{mf}}(d, \eta, \kappa) \), the calculation of which implies solving several optimization problems (5) and (6) (one per failure mode). This type of problem can be solved using decomposition techniques (see Benders, 1962; Geoffrion, 1972) that were applied to reliability optimization problems by Mínguez (2003), and Mínguez et al. 2005, in press). The price that has to be paid for such a simplification is iteration. That is, instead of solving the original problem at once, two simpler problems are solved iteratively: a so-called master problem, which is a problem similar to the original one but replacing the probabilities of failure for each failure mode by linear approximations, and a subproblem or subproblems (one for each failure mode) where the linear approximations of the probabilities of failure are updated for the new design values obtained form the master problem. For a detailed analysis of decomposition techniques see Conejo et al. (in press). For other
alternative techniques for solving similar problems, see Arora (1989).

The following iterative scheme, which solves two optimization problems (the master problem and the subproblems) can be applied to solve the problem (12)–(15):

• Step 0: Initialization. Initialize the iteration counter \( v = 1 \), select some initial values for the design variables \( \bar{d} = \bar{d}_1 \) and evaluate the initial/construction cost \( C_0(\bar{d}_1, \bar{\eta}) \). To improve convergence it is convenient for the initial design to satisfy the failure rate requirements (13).

• Step 1: Subproblem solution. Solve the subproblems, i.e., the problems (5) and (6) modified to

\[
\begin{align*}
\bar{r}_m(\bar{d}, \bar{\eta}, \kappa) &= \text{Maximum } r_m(\bar{d}, \eta, \psi) = r_m(\bar{d}, \eta, \psi) = -\sqrt{x^2 + \kappa} \\
\end{align*}
\]

subject to

\[
\begin{align*}
z &= g_m(d, \eta, \psi), \\
g_m(d, \eta, \psi) &= 0 \\
q(d, \eta) &= \psi \\
\bar{d} &= \bar{d}_r : \mu_{mv}
\end{align*}
\]

where \( \mu_{mv} \) are the corresponding dual variables.\(^3\)

• Step 2: Master problem solution. Update the iteration counter \( v = v + 1 \) and solve the master problem, which consists of replacing the yearly failure rates by linear approximations in problem (12)–(15), that is:

Minimize \( C_0(\bar{d}, \bar{\eta}) \),

subject to

\[
\begin{align*}
r_m^*(\bar{d}, \bar{\eta}, \kappa) \leq R_m^0; \quad m = 1, ..., M \\
n(\bar{d}, \bar{\eta}) = \psi \\
h(\bar{d}, \bar{\eta}, \psi) \leq 0 \\
r_m^*(\bar{d}, \bar{\eta}, \kappa) = r_m(\bar{d}_r - 1, \bar{\eta}, \kappa) + \mu_{mv}^*(\bar{d} - \bar{d}_{r-1}); \quad m = 1, ..., M
\end{align*}
\]

obtaining \( \bar{d}_r \) and \( C_0^*(\bar{d}_r, \bar{\eta}) \). Note that \( r_m^*(\bar{d}, \bar{\eta}, \kappa) \) is a linear approximation of the yearly failure rate \( m \).

• Step 3: Convergence checking. If \( \left| C_0(\bar{d}_r, \bar{\eta}) - C_0(\bar{d}_{r-1}, \bar{\eta}) \right| \) is lower than the tolerance, the procedure stops; otherwise, go to Step 2.

The process is repeated until there is convergence. Observe also that approximating hyperplanes (E.10) are constructed using the partial derivatives of the yearly failure rates (\( \mu_{mv} \)) with respect to the design variables (\( \bar{d} \)).

Appendix F. Sensitivity analysis

The problem of sensitivity analysis in reliability based optimization has been discussed by several authors, see, for example, Enevoldsen (1994), Sorensen and Enevoldsen (1992), Miguez (2003), or Miguez et al. (2004). In this section we show how the duality methods can be applied to sensitivity analysis in a straightforward manner. We emphasize here that the method to be presented in this section is of general validity.

In the problem (E.6)–(E.10) it is very easy to obtain the sensitivities of the optimal initial/construction cost (the objective function) with respect to the failure rate bounds \( R_m^0 \) because they appear on the right hand side of constraint (E.7). When this happens, this sensitivity is simply the value of the dual variable associated with that constraint, which practically all software optimization packages offer for free because it is very easy to calculate once the optimal solution has been found.
The problem arises when the data or parameters with respect to which we want to calculate the sensitivities do not appear on the right hand side of a constraint.

The way of solving this problem consists of generating artificial (redundant) constraints that satisfy such a condition. One way of generating these constraints consists of transforming all the parameters or data with respect to which we desire the sensitivities, into artificial variables and adding the constraints that set the variables to their actual values. To illustrate, we apply this technique to the optimization problems (E.1)–(E.5) and (E.6)–(E.10) at the optimal solution \( \bar{d}^* \).

The problems (E.1)–(E.5) is obviously equivalent to the problem

\[
\text{Maximum } d, \eta^* \text{ subject to } \text{Eq. } (12) \quad \text{(F.1)}
\]

subject to

\[
z = G(d, \eta, \psi, \eta^*, \kappa^*, \bar{d}^*) \quad \text{(F.2)}
\]

\[
q(d, \eta) = \psi \quad \text{(F.3)}
\]

\[
g_m(d, \eta, \psi) = 0 \quad \text{(F.4)}
\]

\[
d^* = \bar{d}^* : \mu_m \quad \text{(F.5)}
\]

\[
\eta^* = \tilde{\eta} : \delta_m \quad \text{(F.6)}
\]

\[
\kappa^* = \kappa : \xi_m, \quad \text{(F.7)}
\]

where \( \bar{d}^*, \eta^* \) and \( \kappa^* \) are the artificial variables.

The basic idea is simple. Assume that we wish to know the sensitivity of the objective function to changes in some data values \( \bar{d}^*, \tilde{\eta} \) and \( \kappa \). Converting the data into artificial variables, \( \bar{d}^*, \eta^* \) and \( \kappa^* \), and setting them, by means of constraints (F.5)–(F.7), to their actual values \( \bar{d}^*, \tilde{\eta} \) and \( \kappa \), we obtain a problem that is equivalent to the initial optimization problem but has a constraint such that the values of the dual variables associated with them give the desired sensitivities. More precisely, the values of the dual variables \( \mu_m, \delta_m \) and \( \xi_m \) associated with constraints (F.5)–(F.7) give the sensitivities of the probability of failure (failure rate) with respect to \( \bar{d}^*, \tilde{\eta} \) and \( \kappa \), respectively.

These sensitivities allow us to determine how the reliability of the breakwater changes when its design values and the statistical parameters of the random variables involved are modified.

Similarly, the problem (E.6)–(E.10) is obviously equivalent to the problem

\[
\text{Minimize } C_0(\bar{d}, \eta^*), \quad \text{(F.8)}
\]

i.e., minimize with respect to \( \bar{d} \), subject to the yearly failure rate and geometric constraints:

\[
r_m^*(\bar{d}, \eta^*, \kappa^*) \leq R_m^0 \quad m = 1, \ldots, M \quad \text{(F.9)}
\]

\[
q(\bar{d}, \eta^*) = \psi \quad \text{(F.10)}
\]

\[
h(\bar{d}, \eta^*, \psi) \leq 0 \quad \text{(F.11)}
\]

\[
r_m^*(\bar{d}, \eta^*, \kappa^*) = r_m(\bar{d}^*, \eta^*, \kappa^*) + \mu_{m-1}^T(\bar{d} - \bar{d}^*) \quad \text{(F.12)}
\]

\[
+ \delta_{m-1}^T(\eta^* - \tilde{\eta}) + \xi_{m-1}^T(\kappa^* - \kappa); \quad m = 1, \ldots, M, \quad \text{(F.13)}
\]

\[
\eta^* = \tilde{\eta} \quad \text{(F.14)}
\]

\[
\kappa^* = \kappa, \quad \text{(F.15)}
\]

where now \( \eta^* \) and \( \kappa^* \) are the artificial variables.

The values of the dual variables associated with constraints (F.9), (F.14) and (F.15) give the sensitivities of the initial/construction cost to \( R_m^0, \tilde{\eta} \) and \( \kappa \), respectively.

These sensitivities allow us to determine how the initial/construction cost of the breakwater changes when the reliability bounds, its geometric dimensions and the statistical parameters of the random variables are modified.

**Remark F.1.** Note that problems (F.1)–(F.7) and (F.8)–(F.15) need to be solved only once, i.e., after solving problem (12) and (15). Because the starting point is already the optimal solution, convergence is ensured at the first iteration.

**Appendix G. Some notations**

- \( a \) coefficient used in the overtopping formula (40) that depends on the structure shape and on the water surface behavior at the seaward face.
- \( a_r \) linear regression coefficient between \( H_{S_{max}} \) and \( H_{max} \).
- \( a_t \) linear regression coefficient between \( T_{z_{max}} \) and \( H_{S_{max}} \).
- \( A \) coefficient in Eq. (16) for modelling the change in the maximum wave height due to random wave breaking.
- \( A_g \) model uncertainty of the horizontal forces for the Goda pressure formula.
- \( A_i \) area of the zone \( i = 1, \ldots, 4 \) shown in Fig. 2.
- \( b \) leeward berm width.
- \( b_o \) coefficient used in the overtopping formula (40) that depends on the structure shape and on the water surface behavior at the seaward face.
- \( b_t \) linear regression coefficient between \( H_{S_{max}} \) and \( H_{max} \).
- \( b_z \) linear regression coefficient between \( T_{z_{max}} \) and \( H_{S_{max}} \).
- \( B \) breakwater caisson width.
- \( B_{se} \) model uncertainty of the vertical forces for the Goda pressure formula.
- \( B_m \) seabed berm width.
- \( B_{z} \) distance of the resultant vertical force component to the harbor side edge.
- \( C \) auxiliary variable defined in Eq. (45).
- \( c_2 \) auxiliary variable defined in Eq. (46).
- \( c_1 \) linear regression coefficient between \( T_{z_{max}} \) and \( H_{S_{max}}, \ H_{max} \).
$C_{ad}$ cost of the armor layer per unit volume.

$C_{ar}$ model uncertainty for the overtopping formula.

$C_{co}$ cost of sand-filled caisson per unit volume.

$C_{co}$ cost of the core per unit volume.

$C_{sp}$ cost of the superstructure concrete per unit volume.

$D$ berm depth in front of the caisson.

$d$ design or geometric variables.

$e$ armor layer thickness.

$F_h$ horizontal force due to water pressure.

$F_{ha}$ horizontal seepage force on the rubble.

$F_m$ model uncertainty parameters related to the different failure modes $m = \{s, b, c, d, r s, o, a\}.$

$F_v$ vertical force due to water pressure.

$g$ acceleration of gravity.

$h$ design water level.

$h_0$ water height at five times $H_{s, max}$ from the breakwater.

$h_1$ water level owing to the astronomical tide.

$h_2$ water level owing to the barometrical or storm surge effects.

$h_b$ concrete crown height.

$h_c$ seaward freeboard.

$h_{lo}$ minimum water depth in front of the breakwater corresponding to the zero port reference level.

$h_n$ core height.

$h_o$ crownwall parapet height.

$h_s$ sand-filled breakwater caisson height.

$h'$ is the submerged height of the crownwall.

$H_{break}$ maximum wave height by breaking conditions.

$H_d$ design wave height.

$H_{max}$ maximum wave height.

$H_{s, max}$ maximum significant wave height.

$I_e$ equivalent cubic block side.

$L$ wave length.

$L_0$ deep water wave length.

$M_c$ moment of the forces acting on the foundation.

$M_h$ moment with respect to $O$ of the horizontal water pressure forces.

$M_s$ moment with respect to $O$ of the vertical water pressure forces.

$N_s$ stability number.

$p_1$ wave pressure at water level.

$p_3$ wave pressure at the caisson’s lower level.

$p_4$ wave pressure at the freeboard level.

$p_{st}$ yearly failure rate for mode $m = \{s, b, c, d, r s, o, a\}.$

$P_a$ uplift pressure on the base of the crownwall.

$q$ mean overtopping volume per unit breakwater length.

$q_0$ maximum mean overtopping volume per unit breakwater length allowed.

$r_{DF}$ distance between the points D and F in Fig. 2.

$R$ dimensionless constant depending on $\gamma_c$ and $\gamma_w$.

$R_m^{0}$ upper bound of yearly failure rate per failure mode $m = \{s, b, c, d, r s, o, a\}.$

$r_{st}$ mean number of storms per year.

$r_m$ yearly failure rate for mode $m = \{s, b, c, d, r s, o, a\}.$

$S_g$ model uncertainty of the seepage horizontal forces for the Goda pressure formula.

$t_r$ tidal range.

$T_{s, max}$ wave period related to the maximum wave height.

$u_1, u_2$ standard uniform random variables.

$v$ standard normal random variable.

$V_{ad}$ armor layer total volume.

$V_{ca}$ sand-filled caisson volume.

$V_{sp}$ concrete superstructure (crown) volume.

$V_{co}$ core total volume.

$V_x$ variation coefficient of the random variable $X.$

$w_o$ caisson parapet width.

$W$ individual armor block weight.

$W_i$ crownwall weight.

$W_i$ weight of zone $i=2, 3, 4$ shown in Fig. 2.

$x$ distance which gives maximum $c_2$ in Eq. (46).

$y$ offset of $W_i$.

$y_{Fs}$ offset of $F_h.$

$y_{Fv}$ offset of $F_v.$

$\alpha_j$ leeward slope angle.

$\alpha_s$ seaward slope angle.

$\beta_m$ reliability factor for mode $m = \{s, b, c, d, r s, o, a\}.$

$\gamma_c$ unit weight of the sand-filled caisson.

$\gamma_w$ water unit weight.

$\gamma_{sp}$ unit weight of the superstructure concrete.

$\gamma_{st}$ scale parameter of the Pareto distribution for $H_{s, max}.$

$\delta_c$ scale parameter of the Weibull distribution for $H_{max}.$

$\eta$ non-optimized design variables.

$\eta$ mean or characteristic value of $\eta.$

$\theta$ angle of the sea bottom.

$\theta_{bs}$ angle between the bottom of the wall and the rupture surface.

$\theta_i$ incidence wave angle.

$\theta_{w}$ incidence wave angle.

$i=0, \ldots , 5,$ auxiliary angles shown in Fig. 2 (f).

$\theta$ parametric vector.

$\theta$ shape parameter of the Pareto distribution for $H_{s, max}.$

$\theta$ shape parameter of the Weibull distribution for $H_{max}.$

$\kappa$ the set of parameters associated with the random variability and dependence structure of the random variables involved.

$\kappa$ shape parameter of the Pareto distribution for $H_{s, max}.$

$\kappa$ shape parameter of the Weibull distribution for friction factor between concrete structure and rubble foundation.

$\mu$ mean value of the random variable $X.$

$\mu_c$ correlation coefficient between $F_h$ and $M_c.$

$\mu_x$ correlation coefficient between $F_v$ and $M_h.$

$\rho$ standard deviation of the random variable $X.$

$\rho_{\delta_c}$ reduced effective internal friction angle of rubble.

$\rho_{\delta_w}$ reduced effective internal friction angle of sand.

$\rho_{\phi_{e}}$ effective internal friction angle of rubble.

$\phi_{e}$ dilation angle of rubble.

$\phi_{s}$ dilation angle of sand.

$\psi_{e}$ the auxiliary (non-basic) variables the values of which can be obtained from those of the basic variables.


