Hydrodynamics and transport in estuaries and rivers by the CBS finite element method

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SUMMARY

This paper presents an extension of the numerical solution of the shallow water equations by the characteristic split (CBS) finite element method to hydraulic engineering problems in estuaries and rivers. A nearly implicit version is formulated and tested and an efficient procedure to implement practical open boundary conditions in the CBS is proposed. This work includes a real numerical application for long term prediction of water quality in terms of salinity concentration in an estuary/river system to improve farmland irrigation given the detailed tidal/fresh water dynamics involved. Copyright © 2006 John Wiley & Sons, Ltd.

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1. INTRODUCTION

Numerical simulation of tidal and fresh water currents in estuaries and rivers, and the corresponding transported quantities, has many interesting applications in fields such as environmental hydraulics, navigation and agriculture. In agriculture, where salinity is a significant transported quantity, the numerical prediction of salt concentration is useful, e.g. to plan reservoir discharges in terms of irrigated farmland demands. The depth integrated shallow water equations and the coupled salinity transport equation govern the hydrodynamics and salinity
transport in (not strongly stratified) estuaries and rivers, and a suitable numerical solution of these PDEs is the CBS finite element algorithm [1, 2].

For long term simulations of low Froude number flows a critical time step in terms of the flow velocity (given by the semiexplicit version of the CBS) is crucial for computational savings. In this work this property has been retained when viscous terms generate more stringent critical timesteps to the solution, thanks to a nearly implicit extension of the algorithm for transport equations. Furthermore, the split of the pressure type terms permits a rational application of the Galerkin method along characteristic transport paths for the intermediate momentum equations and a simple extension for scalar transport equations. Hence, competitive results can be obtained by the Characteristic Galerkin procedure for highly non-linear dynamics (e.g. bore propagation or high velocity channel flows [1]).

The computational domain for estuary/river region is partially confined by an artificial boundary connecting the area with the open sea. For the special open boundary conditions required by the artificial boundary [3–5] a simple first-order radiation condition is adopted [6]. This facilitates a straightforward implementation in the CBS methodology, yields to global conservative approximation, and reduces efficiently spurious reflected waves often produced in the numerical analysis of tidal flows in intricate domains. Some relevant features of a real tidal/fresh water flow numerical analysis of the Guadalquivir River (Spain) are described, to illustrate both the practical use and the properties of the model.

The remainder of this paper is organized as follows. In Section 2 a brief on the analytic model for the fluid and salt transport is presented. The formulation of the numerical model for the PDEs by the CBS method and the implementation of the boundary conditions are outlined in Section 3. The real application of hydrodynamics and salinity transport is discussed in Section 4 and some conclusions in Section 5 complete the paper.

2. SHALLOW WATER AND TRANSPORT EQUATIONS

Shallow Water Equations in the depth integrated form can be written using the summation convention as

\[ \frac{\partial h}{\partial t} + \frac{\partial U_i}{\partial x_i} = 0 \]  
\[ \frac{\partial U_i}{\partial t} + \frac{\partial F_{ij}}{\partial x_j} + \frac{\partial p}{\partial x_i} + Q_i + Q_{\rho_i} = 0 \]

Here \( i, j = 1, 2 \); \( U \) and \( h \), the standard hydrodynamics variables are, respectively, \( U_i = hu_i \) where \( u_i \) is the \( i \) component of the average velocity over the depth and \( h \) the total height of water; \( F_{ij} = hu_iu_j \) is the \( i \) component of the \( j \) flux vector and \( p \) (as a pressure type variable) is

\[ p = \frac{1}{2} g (h^2 - H^2) \]

where \( H \) is the depth of water measured from an arbitrary horizontal reference level. The variables \( h \) and \( H \) are related by: \( h = H + \eta \), where \( \eta \) is the elevation of the free surface.
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with respect to the arbitrary horizontal level. In Equation (2) $Q_i$ denotes the $i$ component of a source vector given by

$$Q_i = -g(h - H) \frac{\partial H}{\partial x_i} + g \frac{u_i |u|}{C^2 h} + r_i - \tau_i \tag{4}$$

The depth integrated source terms in Equation (4) come, respectively, from bottom slope, bottom friction (Chezy Manning formula), Coriolis force: $r_1 = -fu_2$, $r_2 = fu_1$ where $f$ is the Coriolis coefficient ($f = 2\Omega \sin \theta$, $\theta$: latitude of the fluid element and $\Omega = 7 \times 10^{-5} \text{s}^{-1}$ for the earth), and wind tractions $\tau_i$. An additional term in Equation (2) written as $Q_{\rho_i}$ is the $i$ component of a source in case of not well mixed dynamics, where the spatial variation of the density is taken into account as

$$Q_{\rho_i} = \frac{1}{2} gh^2 \frac{\partial (\ln \rho)}{\partial x_i} \tag{5}$$

where the density is $\rho = \rho(s)$, and $s = s(x_i,t)$, $(i = 1, 2)$ is the salinity concentration determined by the salinity transport equation

$$\frac{\partial S}{\partial t} + \frac{\partial (u_i S)}{\partial x_i} - \frac{\partial (h \Phi_i)}{\partial x_i} + R = 0 \tag{6}$$

Here $S = s \cdot h$, $\Phi$ denotes diffusive fluxes, represented in this work by an anisotropic eddy diffusivity model, and $R$ is a source.

### 3. A SPLIT METHOD BASED ON CHARACTERISTICS

The time discretization for Equations (1) and (2) is constructed by proceeding along the characteristic transport path\textsuperscript{4} and is written as follows

$$\frac{1}{c^2} \frac{\Delta p}{\Delta t} + \frac{\partial U_i^n}{\partial x_i} + \frac{\partial (\Delta U_i)}{\partial x_i} = 0 \tag{7}$$

$$\frac{\Delta U_i}{\Delta t} = - \left[ \frac{\partial F_{ij}}{\partial x_j} + Q_i + Q_{\rho_i} \right] + \frac{\Delta t}{2} \left[ u_k \frac{\partial}{\partial x_k} \left( \frac{\partial F_{ij}}{\partial x_j} + Q_i + Q_{\rho_i} \right) \right] - \frac{\partial p^{n+\theta_2}}{\partial x_i} \tag{8}$$

where

$$\frac{\partial p^{n+\theta_2}}{\partial x_i} = (1 - \theta_2) \frac{\partial p^n}{\partial x_i} + \theta_2 \frac{\partial p^{n+1}}{\partial x_i} - (1 - \theta_2) \frac{\Delta t}{2} u_k \frac{\partial}{\partial x_k} \left( \frac{\partial p^n}{\partial x_i} \right) \tag{9}$$

$(i, j, k = 1, 2)$, $\Delta U_i$ and $\Delta p$ indicates the increments of the variables over a timestep $\Delta t$ and the wave celerity $c$ for long waves relates $p$ with the total height of water as

$$c^2 = \frac{dp}{dh} = gh \tag{10}$$

\textsuperscript{4}A detailed derivation of the method can be found in References [1, 2].
The time discretization for the scalar transport equation (6) is

$$\frac{\Delta S}{\Delta t} = - \left[ \frac{\partial (u_i S)}{\partial x_i} + R \right] + \frac{\Delta t}{2} \left[ u_k \frac{\partial}{\partial x_k} \left( \frac{\partial (u_i S)}{\partial x_i} + R + \frac{\partial (h \Phi_i)}{\partial x_i} \right) \right] + \left[ \frac{\partial (h \Phi_i)}{\partial x_i} \right]^{n+\theta_3}$$

(11)

where

$$\left[ \frac{\partial (h \Phi_i)}{\partial x_i} \right]^{n+\theta_3} = \left( 1 - \theta_3 \right) \frac{\partial (h \Phi_i)^n}{\partial x_i} + \theta_3 \frac{\partial (h \Phi_i)^{n+1}}{\partial x_i} - (1 - \theta_3) \frac{\Delta t}{2} u_k \frac{\partial}{\partial x_k} \left( \frac{\partial (h \Phi_i)^n}{\partial x_i} \right)$$

and (0 ≤ θ₁, θ₂, θ₃ ≤ 1). The method is completed by the elimination of ΔUᵢ in the discretized continuity equation (7). This elimination is accomplished by replacing the divergence of Equation (8) into Equation (7), leading to a ‘self adjoint’ type equation for the variable p as

$$\frac{1}{c^2} \frac{\Delta p}{\Delta t} + \theta_1 \theta_2 \frac{\partial \Delta p}{\partial x_i} = - \frac{\partial}{\partial x_i} \left( U_i^n + \theta_1 \Delta U_i^n + \theta_1 \Delta t \frac{\partial}{\partial x_i} p^n \right)$$

(12)

The ‘intermediate’ variable ΔUᵢ* represents the first two terms in square brackets of Equation (8), and is determined explicitly. The practical procedure for the computation of pⁿ⁺¹ and Uᵢⁿ⁺¹ (at the time (n + 1)Δt) is accomplished by the following steps: (a) computation of the intermediate variable, (b) computation of Δp and (c) correction of the momentum components by means of the complete momentum equation (8).

3.1. Space discretization

The spatial discretization of the three steps defined in the previous section is now considered. Applying the standard Galerkin procedure to Equation (8) and the Gauss–Green theorem for the first two terms on the RHS of Equation (8), result in

$$\int_{\Omega} N_l \frac{\Delta U_i^*}{\Delta t} \, d\Omega = \left[ - \int_{\Omega} N_l \frac{\partial F_{ij}}{\partial x_j} \, d\Omega - \int_{\Omega} N_l (Q_i + Q_{pi}) \, d\Omega \right. \right.
+ \frac{\Delta t}{2} \int_{\partial \Omega} \frac{\partial}{\partial x_k} (N_l u_k) \frac{\partial F_{ij}}{\partial x_j} \, d\Omega + \frac{\Delta t}{2} \int_{\Omega} N_l u_k \frac{\partial (Q_i + Q_{pi})}{\partial x_k} \, d\Omega
\left. \left. + \frac{\Delta t}{2} \int_{\partial \Omega} N_l u_k \frac{\partial F_{ij}}{\partial x_j} n_k \, d\Gamma \right]^n \right]$$

(13)

where Ω is the flow domain bounded by ∂Ω and (i, j, k = 1, 2). The right-hand side is computed at time n, Nᵢ is the standard shape function associated with node l and nᵢ is the k component of the outward normal of the boundary ∂Ω.

The ‘p computation step’ can be written in terms of h, by using the wave celerity definition (10). The application of the standard Galerkin method and the Gauss theorem to Equation (12)
results in

\[ \int_{\Omega} N^l \frac{\Delta h}{\Delta t} d\Omega + \Delta t \theta_1 \theta_2 \int_{\Omega} \frac{\partial N^l}{\partial x_i} \tilde{c} \frac{\partial (\Delta h)}{\partial x_i} d\Omega \]

\[ = - \int_{\Omega} N^l \frac{\partial N^l}{\partial x_i} d\Omega + \theta_1 \int_{\Omega} \frac{\partial N^l}{\partial x_i} \left( \Delta U_i^n - \Delta t (c^2)^n \frac{\partial h^n}{\partial x_i} \right) d\Omega \]

\[ - \theta_1 \int_{\partial \Omega} N^l \left[ \Delta U_i^n - \Delta t \left( (c^2)^n \frac{\partial h^n}{\partial x_i} + \theta_2 \tilde{c}^2 \frac{\partial (\Delta h)}{\partial x_i} \right) \right] n_i d\Gamma \]  

\[ \text{(14)} \]

where \( \tilde{c} \) is an average value of the celerity over the timestep and \((c^2)^n\) is evaluated at time \( t^n \).

The last step (or velocity correction) is now established by considering Equation (8). After some convenient manipulations of the second-order term of Equation (8) the spatial discretized correction, using \( h \) and the Galerkin method, gives the total increment of momentum

\[ \int_{\Omega} N^l \frac{\Delta U_i}{\Delta t} d\Omega = \int_{\Omega} N^l \frac{\Delta U_i^n}{\Delta t} d\Omega - (1 - \theta_2) \int_{\Omega} N^l (c^2)^n \frac{\partial h^n}{\partial x_i} d\Omega \]

\[ - \theta_2 \int_{\Omega} N^l (c^2)^{n+1} \frac{\partial h^{n+1}}{\partial x_i} d\Omega - (1 - \theta_2) \frac{\Delta t}{2} \int_{\partial \Omega} \frac{\partial N^l}{\partial x_k} u_k \left( (c^2)^n \frac{\partial h^n}{\partial x_i} \right) d\Omega \]

\[ - (1 - \theta_2) \frac{\Delta t}{2} \int_{\partial \Omega} N^l u_k \left( (c^2)^n \frac{\partial h^n}{\partial x_i} \right) d\Gamma \]

\[ + (1 - \theta_2) \frac{\Delta t}{2} \int_{\partial \Omega} N^l u_k \left( (c^2)^n \frac{\partial h^n}{\partial x_i} \right) d\Gamma \]  

\[ \text{(15)} \]

The discrete form of the bottom slope source term (first term in the rhs of Equation (4)) must be identical to the stationary solution of the Shallow Water equations given by \( h = H + \beta \); \( U_i = 0 \) where \( \beta \) is a constant value. This condition is satisfied if \( H \) is given at nodal points and the discretization \( H = N^l H_l \) is used. When a non-conservative form of the shallow water equations is considered, this source term vanishes and the stationary solution is enforced automatically.

### 3.2. Boundary conditions

The boundary conditions considered are:

**Prescribed elevations in the boundary** \( \Gamma_{\eta} \): This condition implies that

\[ \eta = \bar{\eta} \quad \text{on} \quad \Gamma_{\eta} \subset \Gamma \]  

\[ \text{(16)} \]

**Slip boundary in totally reflecting wall** \( \Gamma_w \): The slip condition imposes zero normal velocities along the boundary

\[ U_i \cdot n_i = 0 \quad \text{on} \quad \Gamma_w \subset \Gamma \]  

\[ \text{(17)} \]

**Open boundaries** \( \Gamma_I \): This type of boundary condition prescribes incoming waves while outgoing waves radiate free. A detailed formulation of this boundary condition for the Shallow
Water equations is treated in Reference [3], while for well posedness and some aspects of a proper implementation see, e.g. Reference [4]. Although more sophisticated open conditions for water wave propagation modelling were developed and implemented by the first author [7, 8], a linear form similar to Reference [6] is here proposed for simplicity. The condition is established as follows. If \((\eta_1, U_1)\) and \((\eta_2, U_2)\) define, respectively, the elevations and momentum of the incoming and the outgoing waves on the boundary, and the elevation of the incoming wave \(\eta_1\) is prescribed as \(\eta_1 = \tilde{\eta}\), an approximate relation can be written as

\[
U_1 = c\tilde{\eta}w; \quad \eta_2 = \eta^R; \quad U_2 = c\eta^R n
\]

where \(w = (\cos \alpha, \sin \alpha)\) denotes the incident wave direction \(\alpha\) measured from the \(x\)-axis, \(n\) is the outward normal to the open boundary and \(\eta^R\) is the reflected wave, generally unknown. To have a solvable problem the direction of the outgoing wave is assumed as the normal of the boundary. Now \(\eta^R\) is eliminated giving the following relation:

\[
-U_n + c\eta = c\tilde{\eta}(1 - w \cdot n) \quad \text{on} \quad \Gamma_l \subset \Gamma
\]

where \(U_n\) is the normal component of the momentum.

**Boundary conditions in the CBS methodology:** No conditions are imposed for the \(\Delta U_i^*\) computation. Now, by inspection of the final form of the second step (14), the terms in the square brackets represent the normal component to the boundary of the momentum equation, as an approximation of order \(\Delta t\). Now condition (18) can be replaced into Equation (14) leading to

\[
\int_\Omega N_l^i \frac{\Delta h}{\Delta t} d\Omega + \Delta t t_1 t_2 \int_\Omega \frac{\partial N_l^i}{\partial x_i} c^2 \frac{\partial (\Delta h)}{\partial x_i} d\Omega + \theta_1 \Delta t \int_{\Gamma_l} N_l^i \frac{\Delta h}{\Delta t} d\Gamma = -\int_\Omega N_l^i \frac{\partial U_i}{\partial x_i} d\Omega + \theta_1 \int_\Omega \frac{\partial N_l^i}{\partial x_i} \left( \Delta U_i^* - \Delta t (c^2)^n \frac{\partial h^n}{\partial x_i} \right) d\Omega + \theta_1 \int_{\Gamma_l} N_l^i \Delta r d\Gamma \quad (19)
\]

where \(\Delta r = c(1 - w \cdot n)\Delta\tilde{\eta}\), \(\Delta\tilde{\eta}\) is the increment of the prescribed elevation over the time step \(\Delta t\), and reminding that \(H = H(x_i)\). The additional boundary term on the left-hand side does not affect symmetry and conditioning of the very convenient original self adjoint system, while it is straightforward to show global conservation of the open condition proposed. Otherwise, for the wall boundary condition the last boundary integral of Equation (14) vanishes at \(\Gamma_w\), while prescribed variables are imposed by elimination. The normal component of velocity is prescribed at the correction stage for wall and open conditions, due to the fact that the discrete momentum equation is also satisfied on the boundary [2].

**Open condition for tidal problems:** For tidal problems, \(\Delta r\) (Equation (19)), is computed for each \(k\)th tidal component as

\[
\Delta r_k^l = A_k c(1 - w_k \cdot n) \cdot \left\{ \sin \left[ \frac{\omega_k}{c} (x' \cdot w_k - ct^{n+1} + \beta_k) \right] - \sin \left[ \frac{\omega_k}{c} (x' \cdot w_k - ct^n + \beta_k) \right] \right\}
\]

where \(A_k, w_k, \omega_k\) and \(\beta_k\) are the amplitude, propagating direction, frequency and phase of the \(k\)th component, respectively, while \(c\) is approximated at time \(t^n\). For a node \(l\) \(\Delta r^l\) is

\[
\Delta r^l = \sum_k \Delta r_k^l
\]
Normal velocity to the open boundary is corrected after the third step as

\[ \Delta U_n^l = c^{(n+1)l}(h^{(n+1)l} - H^l) \]

\[ - \sum_k \left\{ A_k c^{(n+1)l}(1 - w_k \cdot n) \cdot \left[ \sin \left( \frac{\theta_k}{c^{(n+1)l}} (x^l \cdot w_k - c t^{n+1} + \beta_k) \right) \right] \right\} \]

\[ - c^{nl}(h^{nl} - H^l) + \sum_k \left\{ A_k c^{nl}(1 - w_k \cdot n) \cdot \left[ \sin \left( \frac{\theta_k}{c^{nl}} (x^l \cdot w_k - c t^n + \beta_k) \right) \right] \right\} \]

The notation indicates computation as nodal values, where \( \Delta U_n^l \) is the normal component of the momentum on the open boundary at node \( l \), \( c^{(n+1)l} \), \( c^{nl} \) are the celerities at node \( l \) at time \( t^{n+1} \) and time \( t^n \), respectively, and \( h^{(n+1)l} \), \( h^{nl} \) are the total depths at node \( l \) at time \( t^{n+1} \) and time \( t^n \), respectively.

3.3. Final discrete form and time integration options

The final discrete form of the numerical procedure is determined as:

(a) Intermediate momentum calculation: This step is performed by using Equation (13). The variables are approximated in space as

\[ U_{in}^l = N_l \cdot U_{i}^{l(n)}; \quad \Delta U_{i}^* = N_l \cdot \Delta U_{i}^{nl} \]

\[ h^n = N_l \cdot h^{l(n)}; \quad H = N_l H^l \]

(b) Second step: The computation of \( h \) is done by means of Equation (19) and according to the following space discretizations:

\[ h^n = N_l \cdot h^{l(n)}; \quad \Delta h = N_l \cdot \Delta h^l \]

\[ U_{in}^l = N_l \cdot U_{i}^{l(n)}; \quad \Delta r = N_l \cdot \Delta r^l \]

where \( \tilde{c} \) and \( c_{nl} \) are evaluated as an average over the element. Solution of Equation (19) in the semiexplicit form (\( \theta_2 > 0 \)) is achieved by a Jacobi preconditioned conjugate gradient method, requiring only the inexpensive updating of the elementwise wave celerity.

(c) Final velocity correction: The correction is calculated by Equation (15) and the approximation

\[ h^n = N_l \cdot h^{l(n)}; \quad h^{n+1} = N_l \cdot h^{l(n+1)} \]

\[ \Delta U_i = N_l \cdot \Delta U_{i}^l; \quad \Delta U_{i}^* = N_l \cdot \Delta U_{i}^{nl} \]

where \( c^n \) and \( c^{n+1} \) are averaged over the element.

The approximation of the intermediate velocity \( \Delta U_i^* \) is conditionally stable. If a one-dimensional pure convection problem is considered, the stability limit is: \( \Delta t_{\text{crit}} \leq \alpha L/|u| \), where \( L \) is the element size, \( \alpha = 1/3 \) for consistent mass matrix and \( \alpha = 1 \) for lumped mass matrix. The second and third step of the method allow either a semi explicit or an explicit version. For the
fully explicit option ($\theta_2 = 0$), the stability limit is similar to that of explicit procedures such as the Taylor–Galerkin method [1], while for a semi implicit version ($0.5 \leq \theta_1 \leq 1$; $0.5 \leq \theta_2 \leq 1$) the stability limit for the complete solution is bounded by the intermediate momentum calculation. The stability limit of the intermediate solution is obviously modified by viscous terms. This case is addressed in the next section for the salinity transport equation solution.

3.4. Salinity transport equation

The increment $\Delta S$ of the salinity concentration is computed by an additional split (e.g. Reference [9]) such that

$$\Delta S = \Delta S_{CS} + \Delta S_D$$

where $\Delta S_{CS}$ and $\Delta S_D$ correspond, respectively, to convection/sources and diffusion terms. By applying the CBS procedure and neglecting terms higher than second-order the resulting spatio-temporal discrete form of Equation (11) is

$$\int_{\Omega} N^l \frac{\Delta S_{CS}}{\Delta t} d\Omega = \left[ - \int_{\Omega} N^l \frac{\partial (u_i S)}{\partial x_i} d\Omega - \int_{\Omega} N^l R d\Omega ight] + \frac{\Delta t}{2} \int_{\partial \Omega} N^l u_k \frac{\partial R}{\partial x_k} d\Omega + \frac{\Delta t}{2} \int_{\partial \Omega} N^l \frac{\partial R}{\partial x_k} d\Omega $$

while the added diffusion step is

$$\int_{\Omega} N^l \frac{\Delta S_D}{\Delta t} d\Omega = -(1 - \theta_3) \left[ \int_{\Omega} \frac{\partial N^l}{\partial x_i} h^{\mathcal{H}} \frac{\partial S}{\partial x_i} d\Omega \right]^{n+1} - \theta_3 \left[ \int_{\Omega} \frac{\partial N^l}{\partial x_i} h^{\mathcal{H}} \frac{\partial S}{\partial x_i} d\Omega \right]^{n+1} - (1 - \theta_3) \left[ \int_{\partial \Omega} N^l h^{\mathcal{H}} \frac{\partial S}{\partial x_i} n_i d\Gamma \right]^{n+1} + \theta_3 \left[ \int_{\partial \Omega} N^l h^{\mathcal{H}} \frac{\partial S}{\partial x_i} n_i d\Gamma \right]^{n+1}$$

where $\mathcal{H} = \mathcal{H}(x_i, t)$ is an anisotropic and inhomogeneous eddy diffusivity coefficient and $\theta_3 > 0$ implies that the implicit computation for viscous term is active. The boundary conditions are imposed by taking into account the following considerations:

1. For wall boundaries the last integral of Equation (20) vanishes because $u_k n_k = 0$ on $\Gamma_w$.
2. By neglecting the last two terms of Equation (21), the condition $\partial S/\partial n = 0$ on $\Gamma$ is imposed, giving reasonable results at wall boundaries in comparison with measurements [10]. Implicit computation with non-zero flux condition is done by approximating $\partial S/\partial n$ in the last term of Equation (21) at time $t^n$.
3. For prescribed values of $s$ or prescribed values of the incident component in the open boundary, the procedure is the same as for the hydrodynamic variables.
4. HYDRODYNAMICS AND SALINITY CONCENTRATION IN THE GUADALQUIVIR RIVER

This section presents the most relevant features of the numerical simulation of tidal and fresh water currents in the Guadalquivir river and estuary (south of Spain) by the CBS model. The objective of the simulation is to produce a practical tool for the prediction of salinity concentration on irrigated farmlands over long term periods (e.g. irrigation periods from February to September were considered here), based on detailed dynamics of the estuary/river system. The domain of the problem (see Figure 1) is defined by the length of the river of more than 100 km from the open sea (Atlantic Ocean), where a typical estuary type subregion is placed, to the fresh water discharge of the Alcala del Rio dam (upper left side of Figure 1). The farmlands, located in the rectangular area displayed in Figure 1, are mainly dedicated to rice which is strongly dependent on the fresh water supplied by the Alcala del Rio dam. In the studied region the formation of stratified wedges is observed during humid periods (less than 2% of the total days of the year) and at a considerable distance from the cultivated areas, while for dry periods the estuarine Richardson number is much lower than the transition ranges from well mixed to strongly stratified wedges [11]. Therefore, stratification is not relevant for the salinity distribution at irrigation inlets/outlets, and the hypotheses assumed by the depth integrated model described in Section 2 are justified.

4.1. Data structure and preprocessing

A 2D computational domain has been produced in terms of two types of topological data: (a) digitized bathymetries, preprocessed in standard manner, and (b) cross-sections corresponding, respectively, to coastal regions and to river regions. Fixed boundaries are assumed, resulting in a minimum depth required to avoid drying areas and in the subsequent modification of the original transversal sections. The relative errors between the real area and the modified area for the 120 profiles composing the river region were reduced by an iterative process up to maximum errors of: 1.4, 0.8 and 2.9% for mean water level (MWL), MWL ± 1 m and MWL ± 2 m, respectively, giving a final area average error for MWL of 0.1%. Although this approximation permits a considerable amount of computational saving, the resulting transversal profiles are not hydraulically equivalent. However, the assumption is appropriate inasmuch as the transversal sections fulfil the condition of a 'wide open channel'. The standard advancing front grid generation technique was modified to include the following restraints: (a) a minimum number of points for all the river cross-sections, (b) a minimum element size of 25 m² as a compromise between a suitable timestep and the flow resolution demanded, (c) an orientation of the elements toward the river main stream directions (assumed as the average of the tangent direction at the boundary in both shores), and (d) the shape of the triangles determined by a stretching proportional to the width of the river. Hence, reduction in the total number of degrees of freedom and adequate timestep value were achieved by the generated mesh with elongated elements. For the estuarine subdomain, a standard two-dimensional triangulation was generated. Next, the mesh has been tested for $M_2$ tidal dynamics requiring amplitude and phase errors less than 10% in comparison with measurements [10]. Localized mesh refinements were performed to reach stable results and to include the wedge motion (detailed in the next subsection). The ensuing grid, depicted at small scale in Figure 2, has 10110 three node triangular elements, 6307 nodes and 2502 sides. Two representative zooms of Figure 2 are shown in Figure 3,
corresponding in the left part of Figure 3 to the discretization of the estuary (from point N0 to point E2 of Figure 2) and in the right part to a portion of the discretization of the river region close to the measurement point E3 (Figure 2), respectively.

Depth interpolation to nodal values also requires two different treatments. First, the bathymetry-type data available for the coastal zone leads to a standard interpolation procedure from bathymetry points to nodes. Second, for river data, structured in cross-sectional areas, each region defined between every pair of neighbouring profiles is divided into non-overlapped subregions defined by pairs of points with the same depth \( H \) in both cross-sections, for intervals \( \Delta H \) (where \( \Delta H \) was chosen as 0.5 m). Two profiles can have different maximum
depth, leaving unmatched points. This is solved by generating the necessary intermediate points contained in the line between the two points of maximum depth. The subregions obtained are either quadrilateral or triangular, where nodal depth values can be computed by a standard search/interpolation procedure.

4.2. Design of the simulations and main results

The boundary conditions applied for the problem are: (a) open boundary condition for the portion of the boundary linking the computational domain with the open sea, represented by the lower limit of the estuarine region of Figure 3; (b) prescribed normal velocity in the upper left end of the mesh (Alcala del Rio Dam), where fresh water discharge represented in Figure 4 is imposed; and (c) totally reflecting wall boundary condition in the rest of the boundary. For transport equation, prescribed concentrations in the open boundary and zero normal fluxes as described in Section 3.4 in the rest of the boundary were adopted.

The open sea boundary condition is adjusted in amplitudes and phases for the 15 main tidal components in terms of measurements at the point referenced as N0 in Figure 2, where non-linear interaction is weak. This allows a maximum error in amplitude and phase for the resultant tide of 5% in comparison with the measurements. The rest of the 68 tide components contributes less than 4% for the total height and were included by extrapolating the parameters of the principal modes. By adopting prescribed concentrations corresponding to the open sea waters on the open boundary, the back and forth of the wedge must be contained in the domain for all values of fresh water discharges by the dam, displayed in Figure 4 for the studied year (1998). The wedge motion has been verified to determine the final position of the open boundary.

The calibration of the friction (by means of the Manning coefficient $n$) is performed by considering the $M_2$ tidal component and the corresponding measurements [10], giving a smooth behaviour of the friction along the river (Figure 5) and a maximum increase of 15% in regions close to the Alcala del Rio Dam (northwestern end, Figure 2). The small increase of friction damps transversal waves reflected on the vertical wall boundaries in some narrow localized areas. Otherwise, for the definition of the spatial distribution of the diffusivity coefficient required by the eddy diffusivity model the approximate distribution for the Guadalquivir river proposed in Reference [10] is adopted. To adapt this data to 1998, measurements of salinity...
in 5 points for the first three months of 1998 were used, yielding to the final distribution represented in Figure 6.

The initial condition for tidal/fresh water currents calculation is established by a previous artificial stage defined by a start at rest with an augmented friction of 20% to damp initial spurious waves, proceeding to a smooth elimination of the artificial friction by an hyperbolic tangent decay function (in approximately 3 $M_2$ tide cycles), and finally, adding all the tide components to reach the initial time for the hydrodynamics simulation, set as 1st January 1998. The definition of the initial condition for salinity concentration by interpolation of the 5 available measurements points is very crude. The procedure adopted in this work is summarized as follows: for 1st January 1998 the initial condition is defined by a constant salinity along the river corresponding to the fresh water discharge concentration (0.25 g NaCl/l), with an exponential transition along 11 km measured from the open sea boundary to reach sea water concentration. This distribution mimics the position of the wedge close to the exterior boundary.
as if a large fresh water flow is imposed. Consistent with January and February regimes, no discharges were allowed during two months while real flows are imposed from March 1st 1998 (see Figure 4), when irrigation demands begin to be relevant. As an illustration of the back motion of the wedge, Figure 7 shows the salinity concentration evolution for 80 $M_2$ tide cycles beginning at the idealized condition mentioned above, considering $M_2$, $S_2$ and $N_2$ modes. Results of Figure 7 give an estimation of the time needed to reach the concentration threshold ($0.8 \text{ g NaCl/l}$) in the irrigation outlets (at an approximate distance of 50 km from the outer boundary) of 50 days after the tide conditions at the beginning of 1998.

The values of the time integration parameters defined in Section 3 are $\theta_1 = \theta_2 = \theta_3 = \frac{1}{2}$, with the resulting limiting timestep after numerical experimentation of $\Delta t \leq 4.9 \text{ s}$ for the complete set of tide components. If the diffusion terms are solved explicitly, the limiting timestep is
of $\Delta t = 0.1$ s for the distribution of diffusivity represented in Figure 6. An estimation of the diffusion error by the implicit solution is computed by comparing concentrations calculated in a set of nodes along the river region with those calculated explicitly for the process described in the previous section, giving an average error of 3.2%. This error is remarkably low for engineering purposes, taking into account that the implicit model provides a timestep of nearly 50 times the explicit limit. Real improvement should be measured by computing time, giving a real acceleration of nearly 34.

The main results are highlighted in Figures 8–14. In Figures 8 and 9 the behaviour of the tide along the river is plotted, where maximum amplitudes and phases are depicted in comparison with measurements for the three main tidal components and for high, medium and low typical tides in 1998. The local maximum error in amplitude of 5% is produced.
in points close to the dam boundary, where the local increment of friction is maximum. In Figures 10 and 11 computed vs measured concentration in two points are depicted, for a fraction of 1998 and for the complete set of tide components, showing a remarkable overall agreement with few discrepancies mainly due to a diffusivity distribution based on a reduced number of measurement points.
In Figures 12–14 computed evolution of salt concentration in 6 points (indicated as N0, N1, 5 km downstream N1, E2, N4, N6 in Figure 2) and in an inlet irrigation point are plotted. At the inlet point a time interval when the threshold is surpassed was chosen. The simulation was adapted as a practical tool such that the user can define discharge criterias along the irrigation periods analysing the subsequent salt concentrations in the inlets of the irrigation channels.
5. CONCLUSIONS

The finite element CBS model for the numerical solution of estuary/river long term flows presented has some remarkable features. The conjunction of a stability limit dependent on the current velocity instead of on the wave celerity with an implicit computation of diffusion terms to retain the convective limiting timestep is particularly useful for long term problems. This situation is fully illustrated by a case where the period of analysis is several months. The model permits a direct coupling of estuary type problems and complex transient river flows, by solving the depth integrated model for both domains and adapting the disparity of river and coastal data structures. A proper calibration of an anisotropic eddy diffusivity based on available measurements and a detailed dynamics simulation lead to a successful salinity transport model. The incorporation of a computationally efficient open boundary condition in the CBS procedure is an additional practical and necessary improvement for real tidal flow calculations. The properties of the model merge to produce an useful tool for real applications in hydraulic engineering involving complex low Froude number long term dynamics.

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