Aspects of confinement in total cross sections through Bloch-Nordsieck soft gluon summation

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An impact parameter representation for soft gluon radiation has been developed to obtain both the initial decrease in \( pp \) as well as the subsequent rise of the total cross-section \( \sigma_{\text{tot}} \) for \( pp \) and \( pf \). The rapid rise from the perturbative jets is tamed into the experimentally observed mild increase by soft gluon emission. Here we focus on the role played by confinement, in particular the need for a singular \( \alpha_s \) in the infra red region.

1. Introduction

Ab initio calculations of hadronic elastic amplitudes and total cross-sections are difficult given our meager understanding of “soft” physics, that is, the non-perturbative and confinement region of QCD. However, general principles such as unitarity and analyticity must hold for finite ranged hadron dynamics since only finite mass hadrons exist as bound states of quark and glue. Over the past two decades [1], we have developed a rather complete formalism for hadronic (and photon-photon) total cross-sections which combines rigorous constraints from analyticity and unitarity within a QCD framework. For example, dominance of the absorptive parts and small angle scatterings along with bounds from unitarity on total cross-sections are readily incorporated in the eikonal formalism. On the other hand, QCD confinement provides us with power law growths which are asymptotically bounded (in energy). It is augmented with perturbative QCD which produces rising cross-sections through min-jets.

Further, QCD provides soft-gluon radiation from partons as a natural mechanism for saturation, i.e. it tames the growth of the cross-sections forcing them to obey the Froissart bound.

The implementation of the various aspects mentioned above is fairly involved. Given the paucity of space, we shall concentrate here on the important (but oft forgotten) role which confinement plays (at least in two ways). While in the quark-gluon phase, QCD is infinite ranged due to massless gluons, it is confinement which gives rise to finite ranged hadrodynamics and to a Regge behavior. This is easily verified in the string picture where massless \( gg \) and \( gg \) pairs both produce linearly rising Regge trajectories leading to power law growths in the cross-sections [2]. These form an important element of the eikonal which must be supplemented by soft-gluon radiation. We shall discuss this aspect in the next section in some detail.

2. Soft gluon radiation

The total cross-section in the eikonal formalism reads

\[
\sigma_{\text{tot}}(s) = 2 \int (d^2 b) \left[ 1 - e^{-\alpha_s(b_s)/2} \right],
\]
where \( n(b,s) \) may be interpreted as the number of inelastic collisions at a given impact parameter \( b \) for a given \( s \) (the square of the CM energy). In writing the above, a common working assumption has already been made, i.e., the eikonal function has been chosen to be real. This amounts to neglecting the real part of the elastic amplitude completely. At extremely high energies, it has been shown that if the Froissart bound is indeed saturated, then the ratio of the real to the imaginary part of the elastic amplitude in the forward direction must go to zero logarithmically [3], i.e.,

\[
\frac{\Re F(s,0)}{\Im m F(s,0)} \rightarrow \frac{\pi}{\ln s}, \quad as \quad s \rightarrow \infty.
\]  

(2)

Thus, the approximation can be rigorously justified near the forward direction. Since the whole impact parameter picture is based on the assumption that the elastic scattering is concentrated near the forward direction, the assumption about the dominance of the imaginary over the real part is doubly justified.

The simplest model for this eikonal is the eikonal minijet model (EMM) with [4]

\[
n(b,s) = A_{FF}(b)[\sigma_{soft} + \sigma_{jet}],
\]

(3)

where the \( b \)-distribution is a folding of the EM form factors (FF) of the scattering hadrons, \( a, b \)

\[
A_{FF}(b) = \int \frac{d^2 b}{(2\pi)^2} e^{i q \cdot b} F_a(q) F_b(q).
\]

(4)

In FF models, if one arranges for the rise to begin at the correct \( s \), the jet cross-section rises too strongly, as shown in Fig. 1 from [1,5].

We have shown [5] that the reason for this too fast rise is the absence of a diffusive peak. Inclusion of soft radiation softens the rise. It is implemented as follows.

The eikonal function is decomposed into a “soft” and a “hard” piece with their own soft gluon radiation functions

\[
n(b,s) = n_{soft} + n_{hard},
\]

(5)

where

\[
n_{soft} = A_{BN}^{soft} \sigma_{soft}; \quad n_{hard} = A_{BN}^{hard} \sigma_{jet}.
\]

(6)

Figure 1. Total pp and \( p\bar{p} \) cross-sections data compared with a mini-jet model for different values of the minimum jet cut-off \( p_{tmin} \).

with properly normalized Bloch-Nordsieck (BN) functions

\[
A_{BN} = \frac{e^{-h(b,s)}}{\int (d^2 b') e^{-h(b,s)}},
\]

(7)

where

\[
h(b,s) = \int_0^M \frac{dk}{k} \alpha_s(k^2) [1 - J_0(kb)] g(k,M).
\]

(8)

and

\[
g(k,M) = \left( \frac{16}{3\pi} \right) \ln \left( \frac{M + \sqrt{M^2 - k^2}}{k} \right).
\]

(9)

Here \( M \) establishes the scale regulating the maximum energy allowed to a soft gluon in a soft or a hard collision [6]

\[
M = \sqrt{\pi \int (dx_1 dx_2) \int_{z_{min}}^1 dz (1 - z) D(x_1,x_2)}
\]

(10)

where \( D \) denotes the usual quark density expression

\[
D(x_1,x_2) = \sum_{i j} [f_{ji}(x_1)/x_1][f_{ij}(x_2)/x_2].
\]

(11)

The parameter \( p_{tmin} \) occurring in the lower limit \( z_{min} = 4p_{tmin}/(sx_1x_2) \), provides the scale which
characterizes the onset of hard parton-parton scattering. For any parton-parton subprocess with $p_{\text{min}} \approx 1 - 2 \text{ GeV}$, $M$ has a logarithmic increase at reasonably low energy and an almost constant behavior at high energy.

Crucial in this model are the above scales and the behavior of the strong coupling constant which is present in the integral over the soft gluon expression. While in the jet cross-section, $\alpha_s$ never plunges into the infra-red region as the scattering partons are by construction semi-hard, in the soft-gluon spectrum the opposite is true and a regulator is mandatory. We notice that here - as in other soft physics - what primarily matters most is not the value $\alpha_s(0)$ but rather its integral $\int \alpha_s$. We require only that $\alpha_s$ be integrable albeit singular. In our work we employ

$$\alpha_s(k) = \left(\frac{12\pi}{27}\right) \frac{p}{\ln(1 + p(e_k^2)^{2p})},$$

(12)

with $p \approx 3/4$. A different possibility is to use the “frozen” $\alpha_s$ model

$$\alpha_s(k; \text{frozen}) = \left(\frac{12\pi}{27}\right) \frac{1}{\ln(k^2/A^2 + a^2)},$$

(13)

with $a$ a constant. Depending on the value for the constant $a$, these two expressions may lead to very different large-$b$ behavior of the function $n(b, s)$, thereby giving quite distinct $s$-dependence in the rising region of the total proton cross-section. We illustrate this in Fig. 2 where the eikonal mini-jet model EMM has been modified to include - in the hard part - soft gluon initial state radiation from valence quarks [9]. In this figure, the soft part of the cross-section is taken to be the same as in Fig. 1, namely the $b$-dependence is from Eq. (4) and $\sigma_{\text{soft}}$ has been parametrized to reproduce the data.

3. Numerical computations and results

Through Fig. 1 it becomes obvious that some mechanism must be found to modulate the rise of the cross section. It is not unreasonable to suppose that during the transition between the soft and hard region, a significant role is played by the IR singularity in $\alpha_s$. It is natural if the commonly held view is indeed correct that confinement is due to infra-red slavery, which is being modeled here through a singular $\alpha_s$.

We present some evidence in its support by comparing results for different values of the parameter $p$ which quantifies the order of this singularity. For the soft gluon radiation integrals to converge in the IR limit, $p < 1$, which excludes the Richardson limit [10] of 1. The parameter $p$ controls the large $b$-behavior of $n$ and hence the diffraction peak.

Another parameter which plays a major role is the maximum energy $M$ which soft gluons can have for the soft and the hard part. We expect and indeed find that $M$ is limited to about 240 MeV (corresponding to the “size” of the proton) for the soft part of the radiation, whereas for the hard component $M$ continues to rise slowly with energy. This is seen in Fig. 3. Here we have abandoned the previous, power-like parametrization of $\sigma_{\text{soft}}$ in favour of (justification shall follow...
momentarily)

\[ \sigma_{\text{soft}}^{\text{pp}} = \text{constant} \equiv \sigma_0, \quad \sigma_{\text{soft}}^{\text{pp}} = \sigma_0 \left(1 + \frac{2}{\sqrt{s}} \right) \tag{14} \]

with \( \sigma_0 \approx 50 \text{ mb} \), and we have used Eq. (7) instead of Eq. (4), also for the soft-part.

Actually, it is possible to justify where the two components, a Pomeron and the Regge terms, come from. The need for the above choice of \( \sigma_{\text{soft}} \), with two components, the relationship between two power laws as well as the magnitudes of the two terms [2,5] can be found in the hadronic string picture. We shall discuss it here very briefly.

In the string picture of QCD, there are two central excitations which are relevant to hadrons made of light quarks: massless \( q\bar{q} \) and \( gg \) pairs. For these, the energy is given by a sum of three terms: (i) the rotational energy, (ii) the Coulomb energy and (iii) the “confining” energy. If we accept the Wilson area conjecture in QCD, (ii) reduces to the linear potential. Explicitly, in the CM frame of two massless, either a \( q\bar{q} \) or a \( gg \) pair, separated by a relative distance \( r \), with relative angular momentum \( J \), the energy is given by

\[ E_i(J,r) = \frac{2J}{r} - \frac{C_i\hat{\alpha}}{r} + C_i\tau r, \tag{15} \]

where \( i = 1 \) refers to \( q\bar{q} \), \( i = 2 \) refers to \( gg \), \( \tau \) is the “string tension” and the Casimir’s are \( C_1 = C_F = 4/3, C_2 = C_G = 3 \). \( \hat{\alpha} \) is the QCD coupling constant whose value will disappear in the ratio to be considered. The hadronic rest mass for a state of angular momentum \( J \) is then computed through minimising the above energy

\[ M_i(J) = \min_r \left[ \frac{2J}{r} - \frac{C_i\hat{\alpha}}{r} + C_i\tau r \right], \tag{16} \]

which gives

\[ M_i(J) = 2\sqrt{(C_i\tau)[2J - C_i\hat{\alpha}]]. \tag{17} \]

The result may then be inverted to obtain the two sets of linear Regge trajectories \((\alpha_i(s)) \) (not the coupling constant)

\[ \alpha_i(s) = \frac{C_i\hat{\alpha}}{2} + \left(\frac{1}{8C_i\tau}\right)s = \alpha_i(0) + \alpha_i's. \tag{18} \]

Thus, the ratio of the intercepts is given by

\[ \frac{\alpha_{gg}(0)}{\alpha_{qq}(0)} = C_G / C_F = \frac{9}{4}. \tag{19} \]

If we take for the Regge intercept \( \eta \approx 0.48 - 0.5 \), we obtain for \( \varepsilon \approx 0.88 - 0.12 \), a very satisfactory value. Even though we shall not need for total cross-section calculations, we record here that for the slopes we find

\[ \frac{\alpha_{gg}'}{\alpha_{qq}'} = C_F / C_G = \frac{4}{9}. \tag{20} \]

If we take for the Regge slope \( \alpha_{gg}' \approx 0.88 - 0.90 \), we get for \( \alpha_{pp}' \approx 0.39 - 0.40 \), which is fairly satisfactory.

We are not aware of any alternative explanation for these facts: neither for the need of two components nor for the ratio of the two intercepts. In other words, we can justify the well known Donnachie-Landshoff parametrisation [11] in a rather natural way.

With our optimal choice of parameters, we exhibit in Fig. 4, a complete description of \( pp \) and \( pp \) data which includes all effects.

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Figure 3. Total \(pp\) and \(p\bar{p}\) cross-sections data compared with the BN model for both soft and hard part of the cross-section, exhibiting their dependence on the singularity parameter \(p\).


Figure 4. Total \(pp\) and \(p\bar{p}\) cross-sections data compared with the BN model for soft and hard part of the cross-section with a best fit of parameters.