Effects on visual function of approximations of the corneal-ablation profile during refractive surgery

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We offer an analysis that shows that the approximations made for the ablation depth during practical refractive surgery, in which the square-root terms are replaced by the first two terms of the series expansion, can limit the visual function of the observer by reducing the modulation transfer function (MTF). To simulate the refractive-surgical operation, we considered two groups of myopic patients with different ametropia who were emmetropized with different ablation profiles. We made the MTF calculations by taking the spherical aberration into account. In addition, a fuller analysis showed that these approximations limit the possibility of considering surfaces that are aspherical for reshaping the anterior cornea to optimize the observer’s visual function. © 2001 Optical Society of America

1. Introduction

Refractive surgery is intended to make the eye emmetropic by modifying the radius of curvature of the anterior cornea from the ablation on it. In this process, from an optical standpoint, a number of considerations are made concerning the anterior cornea and the corneal region that is ablated. However, the anterior surface of the cornea is assumed to be spherical both before and after surgery, and thus the nonspherical character of the cornea is not considered. The volume eliminated is deemed equivalent to that of a small contact lens, the power of which is calculated from the thin-lens equation. With this equation one calculates the radius of the anterior cornea after surgery, , by relating the magnitude of this radius to the value, in diopters, of the amount of correction required, according to the expression

\[ D = \frac{\Delta n}{R_2} - \frac{\Delta n}{R_1}, \]  

where \( D \) is the value of the correction power, \( \Delta n \) is the difference in refractive index at the air–cornea boundary (\( \Delta n = 0.376 \)), and \( R_1 \) is the radius of the anterior cornea before refractive surgery. We know that Eq. (1) is not exact because it causes some defocusing for high myopia when the thin-lens equation is not valid. If the diameter of the cornea on which the ablation is to be performed is \( d \), the ablation depth \( s(y) \) for a value of height \( y \) over the axis is given by

\[ s(y) = (R_1^2 - y^2)^{1/2} - (R_2^2 - y^2)^{1/2} \]

\[ + \left( R_2^2 - \frac{d^2}{4} \right)^{1/2} - \left( R_1^2 - \frac{d^2}{4} \right)^{1/2}. \]  

During practical ablation an additional approximation is employed, and the terms of the square roots that determine the ablation depth [Eq. (2)] are replaced by the first two terms of their series expansion:

\[ (1 - a)^{1/2} \approx 1 - a/2, \quad a \ll 1. \]  

Substituting this approximation (\( y, d/2 \ll R_1, R_2 \)) into each square root of Eq. (2), and for \( \Delta n = 0.376 \) (\( 2\Delta n \approx 3/4 \)), we get the expression for the ablation depth:

\[ s(y) \approx \frac{4Dy^2}{3} - \frac{Dd^2}{3}. \]  

With respect to the approximations mentioned above, the assumption of a spherical cornea as used to deduce Eq. (2) limits refractive surgery, because this approach does not consider corneal asphericity, although one could use such a consideration to optimize the observer’s visual function, as some authors have shown. The approximation given by expression

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(4) appears small,\(^1\) but the extent to which it affects some visual parameters of the observer has not been quantified to determine whether the approximation is negligible. That is, the use of one expression rather than another implies that the postsurgical state of the cornea is different, and we know that small variations in the anterior cornea (the surface most responsible for ocular refraction) can influence the parameters of the observer’s vision.\(^4,7,8\)

In this study we theoretically calculated the monochromatic modulation transfer function (MTF) for two groups of patients with average myopia who were emmetropized when the ablation depth was calculated by Eq. (2) or its approximation, expression (4), to determine the influence of this approximation on the image quality of the patient who was operated on. To do this, we took the data on the anterior corneas of real eyes provided by a refractive surgical clinic; the rest of the ocular parameters were taken from the data on a schematic eye.\(^9\) The parameter that we used for evaluating visual function was the MTF, a parameter with which the visual qualities of various surgical processes, such as introduction of intraocular lenses and refractive surgery, are evaluated.\(^6,10,11\)

In the MTF computations the spherical aberration was taken into account.

We show that this approximation cannot be made in the case when corneal asphericity in the refractive-surgical process is assumed because, if this approximation is applied, the ablation depth for a height over the optical axis will not be a function of the corneal asphericity. In addition, the same dependence will result, except for one constant, as when spherical surfaces are considered for the anterior cornea [expression (4)]. Consequently, the possibilities of improving refractive surgery by taking asphericity into account will be reduced in practical surgery if the standard approximation is made in the ablation profile.

### 2. Method

To simulate the refractive-surgery operation, we selected two groups of patients with average myopia (−2D and −4D, respectively), and for this experiment the radii and the corneal asphericities of the eyes of these two groups were selected as an average from the data provided by a topographic keratometer (Model 2000, Eyesys) from an ophthalmologic clinic specializing in refractive surgery (see Table 1). The number of myopes in a group ranged from 30 to 40. As we stated above, the standard ablation algorithm, expression (4), does not consider corneal asphericity, although it is applied to real eyes that do present a degree of asphericity (\(Q\)). In our case, to simulate the real cases most accurately, we took the asphericity and the radius of the anterior cornea provided by the keratometer and applied the ablation algorithm, although that algorithm does assume spherical surfaces. In addition, as we shall see below, data from the schematic eye used also included asphericity.\(^9\)

<table>
<thead>
<tr>
<th>Ocular Component</th>
<th>Degree of Myopia</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>−2D</td>
</tr>
<tr>
<td>Anterior corneal radius (mm)</td>
<td>7.75 ± 0.34</td>
</tr>
<tr>
<td>Corneal asphericity</td>
<td>−0.16 ± 0.05</td>
</tr>
<tr>
<td>Axial length (mm)</td>
<td>24.836</td>
</tr>
</tbody>
</table>

\(^a\)The rest of the ocular parameters (see text) were taken from the model of Navarro et al.\(^9\) The axial length for each group of myopic patients was calculated in each case so that they would present the exact value of the ametropia considered.

Aspherical surfaces are normally modeled as conicoid surfaces, which can be described by the equation\(^5–9\)

\[
x^2 + y^2 + (1 + Q)z^2 - 2zR = 0,
\]

where \(R\) is the radius of curvature and \(Q\) is the asphericity parameter; \(x, y\) and \(z\) are spatial coordinates, where the \(z\) axis is the optical axis. Corneal asphericities of the average myopic groups were consistent with the \(Q\) values given by Carney et al.,\(^5\) who reported that \(Q\) tended to be more positive (in our case, slightly so) as the level of myopia increased.

All the other ocular parameters not involved in the surgery (radii and asphericities of the posterior corneal surface, lens surfaces, corneal and lens thickness, anterior-chamber depth, and refractive indices of the various ocular media), except axial length, were assumed to be identical in the two groups of myopes and were taken from the emmetropic-eye model of Navarro et al.\(^9\) for \(\lambda = 555\) nm. Parameters provided by this model appear to be appropriate for the objective of our study, as we sought to study the on-axis optical performance (tested by computing the monochromatic MTF, taking spherical aberration into account), and it is not necessary to apply wide-angle eye models\(^12\) or the structure of the index gradient.\(^12\) As in previous work,\(^6,8–11\) we assumed that the optical axis coincides with the visual axis. The axial length for each group of myopes was calculated (see Table 1) such that it would represent the exact value of the ametropia considered.\(^6,10\)

As indicated above, we used the monochromatic MTF to evaluate the visual performance of the refractive surgery. This function was determined from the spherical aberration for the infinity point once the myopes were emmetropized. For the ablation we used Eq. (2) and the approximation used in practical surgery, expression (4). Both equations make the subject emmetropic, although, as they have different profiles, the spherical aberration and the MTF also differ. The wave-aberration function (\(W\)) was calculated from the transverse spherical aberration, which in turn was determined by ray tracing. In MTF computations, small eventual defocusings caused by ablation-depth approximations and numerical fittings are also accounted for. We took an ablation radius of 3.5 mm. A number of studies\(^6,9,10,14–16\)
thoroughly describe the method of calculating the spherical aberration and the MTF, and, in Appendix A, we provide a cursory explanation. All the figures include the wave-aberration coefficients.

3. Results and Discussion

Figure 1 shows the MTFs that resulted from the refractive surgery for the group of patients with $-2D$ myopia when we applied Eq. (2), (Fig. 1, solid curve), and its approximation, expression (4), (dashed curve). We studied the MTFs for a pupil radius of 3.5 to increase the effects of the aberrations. The figure shows the strong decrease in the MTF when the series expansion is used in practical surgery; in addition, we found many fluctuations from 30 cycles/deg in the MTF. This decrease is due to the additional defocusing$^{14–16}$ introduced with the approximations used, because the ablation in the center is different [Eq. (2) and expression (4) differ for $y = 0$] and therefore the eye appears defocused (see the $W_{20}$ coefficients in the figure) and the resultant curve has a significant decline in the MTF. This phenomenon is more pronounced in the $-4D$ group and increases as the degree of myopia is increased, as reflected by the fact that Eq. (1) (the thin-lens equation) is valid more often for a low value of myopia.$^{1,3}$ Figure 1 also shows a small defocusing coefficient, $W_{20}$, when we apply Eq. (2) for the $-2D$ group. This smaller coefficient is due to the slight defocusing introduced by Eq. (1) and to minor deviations that result from the numerical fitting used in calculating the aberration coefficients (see Appendix A).

To minimize the effects of the defocusing that are due to the approximations applied, and thereby to be able to determine clearly the influence exerted by the ablation profile on image quality after refractive surgery, we used the original Eq. (2) but carried out the series expansion for only the first two terms. The other two terms, which do not depend on the variable $y$, remained unchanged. In this case, from Eq. (2) and applying expression (3), we arrived at

$$s(y) = R_1 - R_2 - \frac{Dy^2}{2\Delta n} + \left( R_2^2 - \frac{d^2}{4} \right)^{1/2} - \left( R_1^2 - \frac{d^2}{4} \right)^{1/2}.$$  \hspace{1cm} (6)

Thus the effect of defocusing is diminished, as Eqs. (2) and (6) coincide for $y = 0$, because we canceled the defocusing that was due to the approximations of the constant terms. In this way, we now can study the influence of different ablation profiles obtained from the series expansion of the terms that contain the variable $y$, the height over the axis. Now we compare the MTF for the two groups of myopes, when they are emmetropized with Eqs. (2) and (6).

Figures 2 and 3 show the differences in the MTFs. Although the differences between the curves are less,
and we do not detect fluctuations in the curves, these differences are still substantial and significant (see the spherical aberration coefficients in the figures) and increase with the spatial frequency, reaching 18.28% and 19.30% for frequencies of 60 cycles/deg for the −2D and the −4D groups, respectively. We averaged the region of the MTFs in intervals of 0.01 from 20 to 60 cycles/deg, getting average differences of 10.81% and 11.24%, respectively, for the −2D and −4D groups. The magnitude of these differences was similar to those reported in other publications on applied visual aspects, for example, in studies of the variations in the MTF that are due to the bending factor of intraocular lenses. Thus we have demonstrated that the approximation in the algorithm for refractive surgery affects image quality not only by the additional defocusing (greater with increased myopia) but also by the variation in the ablation profile.

We have shown that although some authors indicate that the magnitude of approximation (3) is not large, it can be highly important if we perform our analysis by analyzing a visual parameter such as the MTF. In addition, it should be taken into account that this study, like most theoretical studies of spherical eyes and certain practical visual aspects, considers the spherical aberration. It is expected that, on inclusion of other aberrations, the differences in the ablation profile could have a stronger effect on the contrast sensitivity of the observer. In this sense, Jiménez et al., studying the polychromatic MTF (which takes into account spherical and chromatic aberration), demonstrated that one can optimize the visual function after refractive surgery by considering corneal asphericity (Q).

4. Effect of Series Expansion When Corneal Asphericity in the Ablation Is Considered

The effect of approximation (3) can limit the optimization of refractive surgery if the ablation depth is altered to introduce the corneal asphericity. Some practical works have shown the influence of corneal asphericity on visual function after surgery, and some authors have mentioned the importance of taking into account corneal asphericity in refractive surgery. In this sense, Jiménez et al., demonstrated that, if the algorithm given by Eq. (2) is modified to take corneal asphericity into account, the surgical process can be optimized by permitting a choice in the postoperative curvature (Q) of the cornea to improve some aspects of visual function, such as the polychromatic MTF. This algorithm is the generalization of the one normally used in refractive surgery [Eq. (2)], and, to determine it, we need to bear in mind only that the cornea is described by some initial parameters (R1, Q1) that correspond to the conicoid-surface model given by Eq. (5) and that the postoperative cornea is left with some parameters (R2, Q2), as reflected in Fig. 4. Meanwhile, R2 is fixed by Eq. (1), which makes the eye emmetropic, with the parameter Q2 being free to be chosen such as to optimize any visual function. We deduce the ablation depth, as in the spherical case, by imposing the continuity condition in ±d/2 (the two curves coincide with the ablation limit). In this case, after some calculations, we have

\[
s(y) = \frac{1}{1 + Q_1} \left[ R_1^2 - y^2 (1 + Q_1) \right]^{1/2} - \frac{1}{1 + Q_2} R_2^2 - \frac{d^2}{4} \left[ \frac{1}{1 + Q_2} - \frac{1}{1 + Q_1} \right] \left[ R_1^2 - \frac{d^2}{4} (1 + Q_1) \right]^{1/2}.
\]

Equation (7) coincides with Eq. (2), simply making Q1 = Q2 = 0. Thus applied, the postoperative asphericity factor can be chosen to optimize the visual function. The point is that, if we apply the same series expansion to each of the terms of this equation, the influence of corneal asphericity in the ablation disappears. The series expansion of the root gives

\[
\frac{1}{1 + Q_i} \left[ R_i^2 - (1 + Q_i) x^2 \right]^{1/2} = \frac{R_i}{1 + Q_i} \left[ 1 - \frac{(1 + Q_i) x^2}{2 R_i^2} \right] = \frac{R_i x^2}{1 + Q_i} \frac{x^2}{2 R_i}.
\]

After calculations, we find that the ablation depth is

\[
s(y) = \frac{4 D y^2}{3} - \frac{d^2}{(1 + Q_1)8 R_1} - \frac{d^2}{(1 + Q_2)8 R_2}.
\]
For $Q_1 = Q_2 = 0$ we would have [Eq. (4)]

$$s(y) = \frac{4Dy^2}{3} - \frac{d^2}{8R_1} - \frac{d^2}{8R_2} = \frac{4Dy^2}{3} - \frac{Dd^2}{8\Delta n_1}$$

and, if $Q_1 \approx Q_2$,

$$s(y) = \frac{4Dy^2}{3} - \frac{d^2}{(1 + Q_1)8R_1} - \frac{d^2}{(1 + Q_1)8R_2} = \frac{4Dy^2}{3} - \frac{Dd^2}{3(1 + Q_1)}.$$ (11)

Equation (9) shows the dependence of the ablation depth on a height value $y$, and it is the same ($4Dy^2/3$) as when spherical surfaces, without any explicit mathematical dependence on $Q_1$ or $Q_2$, are taken. This means that, on making the series expansion, we lose the information on the corneal asphericity in the ablation, and this asphericity therefore cannot be used to optimize the visual function, as was demonstrated by Jiménez et al.\(^6\) The only difference in the two algorithms, expression (4) and Eq. (9), is a small constant quantity in the ablation on the axis ($y = 0$), causing, as expected, a different defocusing. Therefore consideration of corneal asphericity leads to the same ablation algorithm, except for inclusion of a constant, as when we omit such considerations in the case of applying the standard series expansion to the square roots.

In other words, if we select postoperative corneal asphericity, regardless of the criterion that we choose (minimization of aberrations, optimization of any visual function, etc.), the asphericity will not improve the visual function if we carry out the standard approximations in the ablation depth.

5. Conclusion
This study has demonstrated that the assumption of some approximations in corneal ablation in refractive surgery diminishes the modulation transfer function. The extraordinary ablation capacity of the new refractive-surgery techniques makes it unnecessary to consider these approximations for practical surgery. In addition, theoretical and practical studies support the incorporation of corneal asphericity information in refractive surgery. One should use new algorithms that take into account presurgery and postsurgery corneal asphericity exactly, without applying the standard series expansion in some terms of the ablation depth, to truly consider corneal asphericity and to optimize the visual performance in the surgical process.

Appendix A
The value of the monochromatic MTF is determined from the optical transfer function\(^{14–16}\) OTF:

$$\text{OTF}(f_x, f_y) = \int_{-\infty}^{\infty} P(\xi + \frac{\lambda d f_x}{2}, \eta + \frac{\lambda d f_y}{2}) P(\xi - \frac{\lambda d f_x}{2}, \eta - \frac{\lambda d f_y}{2}) d\xi d\eta$$

where $f_x$ and $f_y$ are spatial frequencies, $P(\xi, \eta)$ is the generalized pupil function, $\lambda$ is the wavelength, and $d_1$ is the distance of the plane of the pupil to the image plane. For a system with circular symmetry the OTF can be determined as\(^{14–16}\):

$$\text{OTF}(w) = \frac{M}{R_p^2} \int_{w/2}^{R_p} r_w \sqrt{1 - r_w^2} P(R_p\xi, R_p\eta) \times P(r_w^*(\xi - w, \eta)) d\xi d\eta.$$

where $w$ is the spatial frequency; $R_p$ is the radius of the pupil; $M$ is a constant, such as $\text{OTF}(0) = 1$; and the generalized pupil function, $P(\xi, \eta)$, is\(^14\)

$$P(\xi, \eta) = \{\exp[-0.1151R_p^2(\xi^2 + \eta^2)]^{1/2} \exp[ikW_{20} \times (\xi^2 + \eta^2) + W_{40}(\xi^2 + \eta^2)^2 + W_{60}(\xi^2 + \eta^2)^3]\}.$$ (A3)

In the function $P(\xi, \eta)$ there are two terms, which represent the Stiles–Crawford apodizing effect and the effect of aberrations, respectively. Meanwhile, $W_{20}$ is the defocusing coefficient that accounts eventually for an additional shift of focus when one uses the approximations in corneal ablation and the numerical fittings that are used for determining the coefficients and $W_{40}$ and $W_{60}$ are the resultant coefficients of the spherical-wave aberration (the third- and fifth-order spherical aberration coefficients, respectively). We verified that the contribution of higher spherical-aberration terms is negligible.

The wave-aberration function $W$ may be related to the transverse spherical aberration as follows:

$$\delta r = -\frac{R_w}{n} \frac{\partial W}{\partial \xi}, \quad \delta s = -\frac{R_w}{n} \frac{\partial W}{\partial \eta},$$ (A4)

where $R_w$ is the radius of curvature of the reference sphere (which is usually defined as the distance from the exit pupil to the paraxial point image), $(\xi, \eta)$ are Cartesian coordinates in the exit pupil, $n$ is the refractive index at the image space, $W$ is the wave-aberration function, and $\delta r$ and $\delta s$ are the components of the ray aberration at the image plane. For a system with rotational symmetry, we can take $\xi = 0$ and use Eqs. (A4). In this case, introducing

$$W_{20}(\lambda) = 1/2R_p^2C_{20}(\lambda), \quad W_{40}(\lambda) = 1/4R_p^4C_{40}(\lambda),$$
\[ W_{60}(\lambda) = \frac{1}{4R_s^6}C_{60}(\lambda), \quad (A5) \]

we can write Eqs. (A4) as

\[ C_{20}(\lambda)\eta + C_{40}(\lambda)\eta^3 + C_{60}(\lambda)\eta^5 = -n\delta s/R_w. \quad (A6) \]

By ray tracing (1000 rays in each case), we calculated the transversal spherical aberration corresponding to each ray of height \( y \) over the optic axis. The object was located at infinity. With these 1000 data pairs, we adjusted the expression given in Eq. (A6), fitting by using the least-squares method to calculate the coefficients \( C_{20}, C_{40}, \) and \( C_{60}. \) (For more-exhaustive information on this procedure, see Malacara and Malacara\textsuperscript{16}.) With these three values we have the information corresponding to the wave aberration on the generalized pupil function [Eq. (A3)] which we use to calculate each monochromatic OTF [Eq. (A1)]. We verified the programs developed to calculate the MTF by reproducing the MTF correctly for different aberrations\textsuperscript{14} The MTFs were shown in cycles per degree (c/deg) instead of cycles/mm for comprehension in visual terms.\textsuperscript{7}

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References