PRIORITY COMMUNICATION

Two-Dimensional Colloidal Aggregation: Concentration Effects

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Extensive numerical simulations of diffusion-limited (DLCA) and reaction-limited (RLCA) colloidal aggregation in two dimensions were performed to elucidate the concentration dependence of the cluster fractal dimension and of the different average cluster sizes. Both on-lattice and off-lattice simulations were used to check the independence of our results on the simulational algorithms and on the space structure. The range in concentration studied spanned 2.5 orders of magnitude. In the DLCA case and in the flocculation regime, it was found that the fractal dimension shows a linear-type increase with the concentration \( \phi \), following the law: \( d_f = d_{f0} + a\phi^c \). For the on-lattice simulations the fractal dimension in the zero concentration limit, \( d_{f0} \), was 1.451 ± 0.002, while for the off-lattice simulations the same quantity took the value 1.445 ± 0.003. The prefactor \( a \) and exponent \( c \) were for the on-lattice simulations equal to 0.633 ± 0.021 and 1.046 ± 0.032, while for the off-lattice simulations they were 1.005 ± 0.059 and 0.999 ± 0.045, respectively. For the exponents \( z \) and \( z' \), defining the increase of the weight-average (\( S_w(t) \)) and number-average (\( S_n(t) \)) cluster sizes as a function of time, we obtained in the DLCA case the laws: \( z = z_o + b\phi^d \) and \( z' = z'_o + b'\phi^{d'} \). For the on-lattice simulations, \( z_o \), \( b \), and \( d \) were equal to 0.593 ± 0.008, 0.696 ± 0.068, and 0.485 ± 0.048, respectively, while for the off-lattice simulations they were 0.595 ± 0.005, 0.807 ± 0.093, and 0.599 ± 0.051. In the case of the exponent \( z' \), the quantities \( z'_o \), \( b' \), and \( d' \) were, for the on-lattice simulations, equal to 0.615 ± 0.004, 0.814 ± 0.081, and 0.620 ± 0.043, respectively, while for the off-lattice algorithm they took the values 0.598 ± 0.002, 0.855 ± 0.035, and 0.610 ± 0.018. In RLCA we have found again that the fractal dimension, in the aggregation regime, shows a similar linear-type increase with the concentration \( d_f = d_{f0} + a\phi^c \), with \( d_{f0} = 1.560 ± 0.004 \), \( a = 0.342 ± 0.039 \), and \( c = 1.000 ± 0.112 \). In this RLCA case it was not possible to find a straight line in the log–log plots of \( S_w(t) \) and \( S_n(t) \) in the aggregation regime considered, and no exponents \( z \) and \( z' \) were defined. We argue however that for sufficiently long periods of time the cluster averages should tend to those for DLCA and, therefore, their exponents should coincide with \( z \) and \( z' \) of the DLCA case. Finally, we present the bell-shaped master curves for the scaling of the cluster size distribution function and their evolution when the concentration increases, for both the DLCA and RLCA cases.

Key Words: colloidal aggregation; fractal dimension; kinetic exponents.

1. INTRODUCTION

Although the two-dimensional aggregation of colloids is a process much less studied than in three dimensions (1), more and more researchers are starting to pay attention to aggregation processes in planar spaces (2–13). This is usually done by confining the colloidal particles to an interface between two fluids, most commonly the suspension liquid and the air on top of it. It has been found that in this case additional medium-range forces appear between the colloidal particles (14, 15), which probably affect the values of the fractal dimension and the kinetic exponents, making them different from the pure diffusion-limited (DLCA) and reaction-limited (RLCA) colloidal aggregation values. It has also been found that in not very dilute systems there is again a change in those values as compared to those in the infinitely dilute system (16). To discern which changes are due to the concentration effect and which ones are due to the medium-range forces, it appears necessary to have a thorough study of the changes in the mentioned quantities when only the concentration effect is present. In this paper we address this important point by considering extensive two-dimensional computer simulations of colloidal aggregation, in the DLCA and RLCA limits, when only short-ranged adhesion forces at contact are taken into account. The concentration was varied from an area fraction of 0.001 to a maximum fraction of 0.4; this allows us to study the changes in those quantities when the concentration is varied.
Different algorithms were investigated to check the independence of the results on the particular algorithm used. We finally settled with the two algorithms used in this work, one on-lattice and the other off-lattice. The DLCA runs were made with both the on-lattice algorithm and the off-lattice one, while for the RLCA case, where a sticking probability of 0.0005 was used, only the on-lattice algorithm was run due to its computational speed.

This paper is organized in the following form: In section 2 we will give a brief description of the computer models used to study the effect; we will comment also, in the same section, on the size of each simulation and the number of simulations run for each concentration and for each case (DLCA on-lattice or off-lattice or RLCA on-lattice). In section 3 we will discuss the results for the DLCA case. In subsection 3a we will first describe the results for the fractal dimension, in the flocculation regime, as a function of concentration. We will see how the functional form proportioned by both the on-lattice and the off-lattice algorithms coincides, with a minor change arising from the different definitions of the concentrations in those cases. In subsection 3b we present our results for the dynamical exponents $z$ and $z'$, which describe the increase of $S_n(t)$ and $S_s(t)$ as a function of time, respectively. We will see again how the on-lattice and off-lattice algorithms proportion roughly the same functional forms for the dependence on the concentration of the exponents $z$ and $z'$. Subsection 3c ends section 3 with plots of the master curve of the scaled size distribution function for some selected values of the concentration, showing that in both on-lattice and off-lattice simulations the same master curves are obtained. In section 4, for RLCA, a similar thing as in section 3 was done, but only for on-lattice simulations as mentioned. In subsection 4a the results for the fractal dimension are presented and in subsection 4b we discuss the kinetic exponents, while in subsection 4c we present the master curves for the cluster size distribution function. Finally, in section 5 we discuss our results, always making a comparison with the already worked out three-dimensional case (16).

## 2. COMPUTER MODELS

Two were the computer models used in this work; one is an on-lattice model while the other is off-lattice. In the on-lattice algorithm, we consider a two-dimensional square lattice with periodic boundary conditions where some of the cells can be occupied (by a colloidal particle) or can be empty (solvent). Initially, all the colloidal particles are randomly distributed and unaggregated though some of them may touch each other at some points. As the aggregation proceeds, we deal with a collection of clusters made of nearest-neighbor lattice cells that are diffusing randomly. One of the clusters is picked at random and moved by one lattice unit in a random direction, only if a random number $X$ uniformly distributed in the range $0 < X < 1$ satisfies the condition $X < D(s)/D_{max}$, where $D(s) \sim s^{-(d_f+1)}$ is the diffusion coefficient for the selected cluster of size $s$ and $D_{max}$ is the maximum diffusion coefficient for any cluster in the system. Here, $d_f$ is the value of the fractal dimension at the given concentration. Typically, one simulation was done for each concentration to have a rough estimate of $d_f$, whose value was used in all the simulations for that concentration. Once a cluster is selected, the time is incremented by $1/(N_c D_{max})$, where $N_c$ is the number of clusters in the system at that time, independent of whether the cluster has actually moved or not. An encounter is defined by an attempt of our moving cluster to overlap the lattice cells occupied by another. In this case the move is not permitted and the moving cluster either sticks (and is merged) to the other with probability $P_o$ or remains side by side to the other with probability $1 - P_o$. The values used for $P_o$ were 1 for the DLCA case and 0.0005 for RLCA.

In the off-lattice algorithm circular particles with a unit radius are initially also randomly distributed in a square box without particle overlapping. At the beginning of the aggregation particles are thus unaggregated. The movement of the aggregates consists of steps of length $\Delta$ in a randomly selected direction. Randomly picked aggregates are also displaced if a random number $X$ uniformly distributed in the range $0 < X < 1$ satisfies the condition $X < D(s)/D_{max}$; however, in this off-lattice model the time increases as $\Delta^2/(N_c D_{max})$ each time an aggregate movement is attempted. The $\Delta^2$ term is needed to correlate the times of the on-lattice and of the off-lattice models. In the off-lattice model, the diffusion coefficient $D(s)$ of an aggregate was computed from it radius of gyration, $R_g$, using the expression $D(s) \sim 1/R_g$. This assumption is equivalent to the on-lattice one because the dependence of the cluster size with its radius of gyration is given by $s \sim R_g^{d_f}$, but it does not imply the need of an assumption about the fractal dimension value. Periodic boundary conditions are also introduced in the off-lattice model to prevent wall effects on the simulation results. When a moved cluster overlaps another one, its displacement is corrected backward along the same line until both clusters barely touch each other at one point; afterward, they are joined. As we have indicated above, off-lattice simulations were done only for the DLCA case.

Both algorithms proportioned us four files that contained (1) the number of clusters as a function of time, (2) the weight-average cluster size also as a function of time, (3) the cluster size distribution (that is, the number of clusters for the different sizes) at selected times during the aggregation, and (4) the radius of gyration as a function of size for all the clusters formed during the aggregation. In both the on-lattice and the off-lattice algorithms 13 concentrations $\phi$ were considered: 0.001, 0.003, 0.007, 0.01, 0.03, 0.07, 0.1, 0.15, 0.2, 0.25, 0.3, 0.35, and 0.4. In the on-lattice case the $\phi$ was defined as the fraction of occupied cells of the lattice, while in the off-lattice algorithm it was defined as the area fraction occupied by the particles. Table 1 shows the number of particles and the number of simulation runs made for each concentration in each of the 3 cases (DLCA on-lattice, DLCA off-lattice, and RLCA on-lattice). Depending on the concentration, some of the simulations would gel (percolate
TABLE 1
The Size and Number of Computer Runs Made for Each Concentration

<table>
<thead>
<tr>
<th>Density</th>
<th>DLCA off-lattice</th>
<th>DLCA on-lattice</th>
<th>RLCA on-lattice</th>
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</thead>
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<td></td>
<td>No. of runs</td>
<td>No. of particles</td>
<td>No. of runs</td>
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</table>

3. DLCA RESULTS

3.1. The Cluster Fractal Dimension

The fractal dimension of the clusters was obtained via the relation between their radii of gyration and their sizes ($R_g \sim N$). If that relation held for the clusters of all sizes, it would be straightforward to obtain the fractal dimension by just plotting the $R_g$ vs $N$ for all the clusters formed during the aggregation, on a log–log scale, and finding the slope of a fitted straight line that would give us the inverse of the $d_f$. Unfortunately, the situation is not so simple, and it turns out that the above relation does not hold for the very small clusters and also for the larger clusters in the case of the higher concentrations in our finite size simulations. The relation $R_g \sim N$ is an asymptotic equation that should be valid only in the limit of large clusters (large $N$) and that is why it should not be valid for the very small clusters, unless we add additional terms to the relation, the so-called correction to scaling terms. In the case of the higher concentrations in our finite size simulations, at close proximity to the gel point, some of the larger clusters start to “feel” the finite size of the system and their fractal dimension tries to change from the usual DLCA or RLCA fractal dimension in the flocculation regime to the fractal dimension of percolating clusters. As we will see, this obliged us to make the mentioned log–log plot not by drawing a point for each individual cluster formed but instead by taking the average of the radii of gyration over all clusters inside segments of constant magnitude on the logarithmic $N$ scale. This would give us a smooth $R_g$ vs $N$ line, where any curvature describing the departure from the above law can be detected. In Fig. 1 we
illustrate these points by considering a low-concentration and a high-concentration case. In Fig. 1a we are plotting this averaged radius of gyration vs $N$ for a low-concentration value of 0.001. As we can see, there is a very good straight line for all the sizes considered, and the fractal dimension obtained (1.448) is very close to the accepted value for DLCA clusters at infinite dilution (~1.45). In Fig. 1b we are now plotting the $R_g$ vs $N$ for all the clusters formed during the 50 simulations of a concentration of 0.03. We could be tempted to draw a straight line over the data points because the scattering of the data does not allow us to see that there is an actual curvature on the plot. However, in Fig. 1c we now plot the averaged radius of gyration vs $N$ for this same concentration of 0.03. We can now clearly see that there is a quite noticeable curvature for the larger cluster sizes. This indicates that these clusters try to reach the higher fractal dimension of the percolating clusters in two dimensions. The two arrows define the section of the plot for which we can define a straight line, and the value of the fractal dimension obtained in this case was $d_f = 1.464$. We have checked that this curvature is due to the finite size of our samples. To do this, a few simulations were run for the same concentrations at which the curvature was starting to appear ($\phi = 0.01$ and 0.03) but with a linear size of the system of 2 and 4 times the size of the original simulations. It was seen that in this case the section of the plot for which a straight line can be defined (the part between the arrows) widens by the movement of the right arrow further to the right. Therefore, the procedure used to obtain the fractal dimension for the different concentrations was to perform the mentioned average, locate the part of the plot for which a straight line can be defined, and obtain the $d_f$ as the inverse of the slope of this straight line.

In Figs. 2a and 2b we are showing the dependence of the fractal dimension on $\phi$, for the on-lattice and the off-lattice algorithms, respectively. In both cases the increase is approximately linear, starting with a zero concentration DLCA fractal dimension of about 1.45. The law for the increase of the $d_f$ when $\phi$ is varied was found, by a multiparameter fit, to be $d_f = d_{f0} + a \phi^c$. For the on-lattice case we found $d_{f0} = 1.451 \pm 0.002$, $a = 0.633 \pm 0.021$, and $c = 1.046 \pm 0.032$, while for the off-lattice algorithm $d_{f0} = 1.445 \pm 0.003$, $a = 1.005 \pm 0.059$, and $c = 0.999 \pm 0.045$. In the case of the off-lattice algorithm a new effect starts to appear for the two higher concentrations. First, it is clear that the fractal dimension cannot increase to values higher than 2, which is the space dimensionality in which we are working. If we, however, try to continue the straight line all the way up to $d_f = 2.0$, that occurs for a value of $\phi$ approximately equal to 0.55. At these high concentrations we are reaching the percolation threshold for individual particles and the aggregation problem starts to become meaningless. That is why we should stop considering concentrations higher than 0.3 in our calculations, and the multiparameter fit for this off-lattice case did not consider the last two points ($\phi = 0.35$ and 0.40). In the linear laws for the increase of the $d_f$ with $\phi$ the only noticeable difference in the two sets of the parameters was the prefactor $a$ which, given the similarity in the two exponents $c$, indicates that the two concentrations $\phi$ are not equal but are proportional to each other, as far as the fractal dimension is concerned. The relation is $\phi_{on} \approx 1.59 \phi_{off}$. That is why in the on-lattice case we did not see this effect because we would need a concentration of around 0.48 that would correspond to the off-lattice concentration of 0.30, which is the threshold at which the effect starts to appear.

In view of the close proximity to a linear dependence of the cluster fractal dimension as a function of concentration found in this two-dimensional case and of the closeness to a square root dependence in the three-dimensional case (16), it is tempting to consider these as exact results. If this were the case, we could just fit the straight line $d_f = d_{f0} + a \phi$ to the data points to obtain better values for the parameters $d_{f0}$ and $a$. The result of such an analysis is, for the on-lattice case, $d_{f0} = 1.449 \pm 0.002$ and $a = 0.606 \pm 0.008$, while for the off-lattice case, $d_{f0} = 1.445 \pm 0.002$ and $a = 1.006 \pm 0.016$. It is worth noting that the $d_{f0}$ values get closer to each other and that the error bars, particularly for the parameter $a$, tighten appreciably. The relation between the on-lattice and off-lattice concentrations is now $\phi_{on} \approx 1.66 \phi_{off}$.
3.2. The Cluster Size Averages and the Kinetic Exponents $z$ and $z'$

In the case of the weight-average cluster size vs time, a similar thing as with the radius of gyration plots vs size occurred. That is, the log–log plots of the $S_w(t)$ vs $t$ for low concentrations present a wide zone for which there is linear behavior from a certain initial time all the way to the end of the simulation. The slope of the straight line on the log–log plots proportions the kinetic exponent $z$. However, for higher concentrations, that zone shrinks and, for those concentrations for which there is gelation, the $S_w(t)$ speeds up, departing from the straight line on the log–log plots. We have therefore fitted the straight line only inside the time zones for which the straight line is established. The shrinking of the zone becomes more noticeable here than in the case of the fractal dimension and we were not able to obtain the $z$ exponent for all the concentrations used in the present work. In fact, in only the first six concentrations (0.001, 0.003, 0.007, 0.01, 0.03, and 0.07) the straight lines were clear enough to obtain an exponent $z$. In Fig. 3a we are plotting the $\phi$ dependence of the exponent $z$ for the on-lattice simulations; the points correspond to the $z$ values for each of the concentrations while the curve drawn over the data points corresponds to the function $z = z_o + b \phi^d$. In Fig. 3b we are drawing the corresponding plot for the off-lattice simulations. The values of the parameters for the on-lattice algorithm were obtained as $z_o = 0.593 \pm 0.008$, $b = 0.696 \pm 0.068$, and $d = 0.485 \pm 0.048$. For the off-lattice case, we obtained $z_o = 0.595 \pm 0.005$, $b = 0.807 \pm 0.093$, and $d = 0.599 \pm 0.051$.

In a similar way as with the weight-average cluster size, we define the kinetic exponent $z'$ from the number-average cluster size $S_n(t)$. It is clear that this exponent can also be calculated as the negative of the slope obtained from the number of clusters as a function of time, when plotted on a log–log scale, and that is the way we obtained it. Again, in only the first six concentration values the data were able to proportion a clear straight line whose slope was the negative of the exponent $z'$. In Figs. 4a and 4b we are plotting the $z'$ exponent as a function of $\phi$ for the on-lattice and off-lattice cases, respectively. The points were fitted to the functional form $z' = z'_0 + b' \phi^d$.

FIG. 3. The kinetic exponent $z$ obtained from the time dependence of the weight-average size, $S_w(t)$, for DLCA simulations. In Figs. 3a and 3b we plot the $\phi$ dependence of the exponent $z$ for the on-lattice and off-lattice simulations, respectively. The points correspond to the $z$ values for each of the concentrations while the curve drawn over the data points corresponds to the function $z = z_o + b \phi^d$.

FIG. 4. The kinetic exponent $z'$ obtained from the time dependence of the number-average cluster size, $S_n(t)$, for DLCA simulations. In Figs. 4a and 4b we plot the $z'$ exponent as a function of $\phi$ for the on-lattice and off-lattice cases, respectively.
3.3. Scaling of the Cluster Size Distribution Function

We have found a good scaling of the cluster size distribution function, \( N_s(t) \), for all the concentrations studied. Nonetheless, the shape of the master curve obtained when plotting \( f = S_w(t)^2 N_s(t)/N_0 \) vs \( x = s/S_w(t) \), for different times during the aggregation, changes a bit when we vary the concentration. In Fig. 5 we show this fact by plotting the master curves for a low- and high-concentration case, for both the on-lattice and off-lattice simulations. In Fig. 5a, for the concentration \( \phi = 0.007 \), the curve is bell-shaped although a bit asymmetric, with a shorter left arm, while in Fig. 5b, for \( \phi = 0.2 \), the bell widens up somewhat and the left arm becomes even shorter. In both figures the open squares come from the on-lattice simulations while the filled triangles were obtained from the off-lattice ones. It is interesting to note the superposition of both squares and triangles in each figure which indicates that, as far as the dynamics are concerned, the on-lattice and the off-lattice simulations, correspond to each other for the same \( \phi \). This is in agreement with what we said in the last paragraph that the relation between the two \( \phi \)'s is more like an equality as far as the dynamics are concerned. The obtention of basically the same results for the on-lattice and the off-lattice simulations validates and gives support to them, making us expect their results to be equal to those for the real experimental system.

4. RLCA RESULTS

4.1. The Cluster Fractal Dimension

For RLCA, a similar procedure as in the DLCA case was followed to find the concentration dependence of the cluster fractal dimension. To obtain the fractal dimension for a given concentration, we again made the smoothing of the \( R_g \) vs \( N \) curve described in the last section to identify the section behaving as a straight line, obtaining the \( d_f \) as the inverse of the slope of this straight line. In Fig. 6 each of the points corresponds to the \( d_f \)
obtained at the corresponding concentration for the on-lattice simulations. The continuous curve (which actually turned out to be a straight line) was obtained from a multiparameter fit to the points of the type \( d_f = d_0 + a\phi^c \). The fit proportioned the following values of the parameters: \( d_0 = 1.560 \pm 0.004 \), \( a = 0.342 \pm 0.039 \), and \( c = 1.000 \pm 0.112 \). Note that the value \( d_0 = 1.560 \) is close to the accepted value of the RLCA cluster fractal dimension in the dilute limit (\( d_f \approx 1.55 \)). We end this subsection by making a similar analysis to that done for the DLCA case. That is, let us assume that \( c = 1 \) is an exact result and fit the law \( d_f = d_0 + a\phi \) to the data points. The result is \( d_0 = 1.560 \pm 0.003 \) and \( a = 0.342 \pm 0.014 \).

### 4.2. The Behavior of the Cluster Size Averages

It was found that the weight-average cluster size did not present a straight line, when plotted on a log–log scale as a function of time, at least for intermediate times. It was, however, also found that the continuous curvature of \( S_w(t) \) ended, at the final times and for the lowest concentrations, with a value of the slope very close to the exponent \( z \) obtained for the DLCA simulation at the corresponding concentration. Figure 7a illustrates this fact, for a volume fraction of \( \phi = 0.007 \), where the continuous curve is precisely \( S_w(t) \). The broken straight line, with a slope equal to the DLCA \( z \) exponent for the corresponding \( \phi \) (\( z = 0.656 \)), is just a guide to the eye. We can see that indeed the final slope of \( S_w(t) \) is very close to the exponent \( z \). This may be a manifestation of the statement that any RLCA process with a finite sticking probability crosses over, for sufficiently long periods of time, to a DLCA process. It is very possible that the sticking probability we used was too low to be able to detect the full crossover to a straight line with a slope equal to \( z \). This effect has been predicted theoretically by solving the Smoluchowski equation with a nonhomogeneous kernel that takes into account the sticking probability for the aggregates (17).

For higher concentrations, for which we have gelation at early times during the aggregation in our finite size boxes, the percolation effect now comes in and forces the \( S_w(t) \) to increase very rapidly; in fact, it would diverge in an infinite system. In Fig. 7b this behavior is shown, for a concentration of \( \phi = 0.014 \). By trying different values of the exponent, we found that the best collapse of the data was obtained with an exponent equal to 2.1 for all the concentrations studied.

![FIG. 7](image)

FIG. 7. For the RLCA simulations the weight-average cluster size did not present a straight line when plotted on a log–log scale as a function of time, at least for intermediate times. Figures 7a and 7b illustrate this fact for a low concentration of \( \phi = 0.007 \) and a high concentration of \( \phi = 0.25 \), respectively.

### 4.3. Scaling of the Cluster Size Distribution Function

In Fig. 8 we are showing the scaling of the cluster size distribution for low and high concentrations. We again obtain, as in DLCA, bell-shaped curves for all the concentrations studied. The curves are however more asymmetric than in DLCA and a little bit skewed. Another difference from the DLCA case is that the curves do not widen up appreciably as we increase the concentration. In Fig. 8a the master curve obtained by plotting \( f = S_w(t)N_c(t)/N_0 \) vs \( x = s/S_w(t) \) for a concentration of \( \phi = 0.001 \) is shown, while in Fig. 8b we show the same for a much higher concentration of 0.3. It should be pointed out now that the data collapse was very poor when using the usual exponent of 2 for the \( S_w(t) \) in the definition of \( f \). By trying different values of the exponent, we found that the best collapse of the data was obtained with an exponent equal to 2.1 for all the concentrations studied.
5. DISCUSSION

We have seen a real effect in the increase of the cluster fractal dimension as we increase the concentration. This is to be expected because, for a highly dilute system, two clusters touch and merge mainly at the tips of their long arms, the resulting cluster being very open. However, for a higher concentration, the two clusters that are going to touch were born very close to each other, becoming easier for them to touch not at the tips but in the middle. This makes the clusters more compact than those in the dilute case. The increase in the exponent \( z \) as the concentration is increased probably rests in the fact that the clusters need to travel shorter distances to touch and merge. The equality of our results in the DLCA case for both the on-lattice and off-lattice algorithms (with minor differences coming from the different definitions of the volume fractions in both cases) validates our simulations and gives support to our results. It has been found before, in the three-dimensional case [16], an increase in the \( d_f \) with \( \phi \) roughly of a square root type in both the DLCA and RLCA processes. The linear increase we have found here in two dimensions indicates that the exponent \( c \) of \( \phi \), in the law \( d_f = d_{\phi_0} + a \phi^c \), is highly influenced by the dimensionality of the space in which the aggregation is taking place. However, the exponents \( d \) and \( d' \) in the laws \( z = z_0 + b \phi^d \) and \( z' = z'_0 + b' \phi^{d'} \) do not appear to be greatly correlated to the space dimensionality, staying around 0.5–0.6 [16] for the two space dimensionalities. In the case of the fractal dimension, being a structural quantity, it is possible that its value at the higher concentrations is dictated by the efficiency with which we can pack fractal clusters in the space. The slower increase for the three-dimensional case, as a square root type, suggests that we are able to pack more efficiently those clusters in three dimensions than in two dimensions, which in turn may be due to the interpenetrations that occur between clusters in the highly concentrated systems. Therefore, the clusters do not need to compact too much and consequently the fractal dimension does not need to increase as much. These interpenetrations do not occur so easily in two dimensions and the clusters need to become more compact to fit the allotted space, increasing in this way their fractal dimension. On the other hand, for the dynamical quantities the situation is not so different. In both dimensions the clusters and particles are performing random walks, which follow precisely the same law relating the diffusing distance to the time. This may be the reason why there is a similar law for the increase in the exponents \( z \) and \( z' \) with concentration in two and three dimensions.

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