ON REAL HYPERSURFACES WITH
$\eta$-PARALLEL CURVATURE TENSOR
IN COMPLEX SPACE FORMS

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Abstract. The purpose of this paper is to give a complete classification of
real hypersurfaces $M$ in complex space forms $M_n(c)$, $c \neq 0$ in terms of an $\eta$-parallel
curvature tensor and a certain commutative condition defined on the distribution
$T_0 = \{X \in T_o M | X \perp \xi \}$ of $M$ in $M_n(c)$.

1. Introduction

A complex n ($\geq 2$)-dimensional Kaehlerian manifold of constant holomor-
phic sectional curvature $c$ is called a complex space form, which is denoted
by $M_n(c)$. A complete and simply connected complex space form is a com-
plex projective space $P_n(\mathbb{C})$, a complex Euclidean space $\mathbb{C}^n$ or a complex
hyperbolic space $H_n(\mathbb{C})$, according as $c > 0$, $c = 0$ or $c < 0$. The induced al-
most contact metric structure of a real hypersurface $M$ of $M_n(c)$ is denoted
by $(\phi, \xi, \eta, g).

Until now several kinds of real hypersurfaces have been investigated by
many differential geometers from different view points ([4], [6], [7], [11], [14],
[15] and [16]). Among them in a complex projective space $P_n(\mathbb{C})$ Takagi [19]
showed that these hypersurfaces of $P_n(\mathbb{C})$ could be divided into six types
which are said to be of type $A_1$, $A_2$, $B$, $C$, $D$, and $E$, and in [5] Cecil-Ryan
and [10] Kimura proved that they are realized as the tubes of constant radius
over Kaehler submanifolds if the structure vector field $\xi$ is principal.

On the other hand, in [2] and [3] Berndt has called real hypersurfaces
in $M_n(c)$ with the principal structure vector $\xi$ Hopf real hypersurfaces
and has shown that all such kind of real hypersurfaces in complex hyperbolic

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spaces $H_n(C)$ with constant principal curvatures are realized as the tubes of constant radius over certain submanifolds. Nowadays in $H_n(C)$ they are said to be of type $A_0$, $A_1$, $A_2$, and $B$.

It can be easily seen that there does not exist any real hypersurface in $M_n(c)$, $c \neq 0$, which is locally symmetric, that is, $\nabla R = 0$. From this point of view we introduce the notion of $\eta$-parallel curvature tensor which can be defined on the distribution $T_0$ in such a way that

\[(I) \quad g\left( (\nabla_X R)(Y, Z) U, V \right) = 0 \]

for any $X$, $Y$, $Z$, $U$, and $V$ in a distribution $T_0$ orthogonal to $\xi$. This notion is weaker than that of $\eta$-parallel second fundamental tensor $g\left( (\nabla_X A)Y, Z \right) = 0$ for any $X$, $Y$, and $Z \in T_0$. Moreover, the second fundamental tensor of ruled real hypersurfaces constructed by Kimura and Maeda [11] for $c > 0$ and Suh [17] for $c < 0$ is seen to be $\eta$-parallel. So ruled real hypersurfaces naturally satisfy the notion of $\eta$-parallel curvature tensor.

In the paper [1] Baikoussis, Lyu and Suh have considered Hopf real hypersurfaces in $M_n(c)$ with $\eta$-parallel curvature tensor and have shown that they are locally congruent to real hypersurfaces of type $A_1$, $A_2$ and $B$ for $c > 0$ and of type $A_0$, $A_1$, $A_2$ and $B$ for $c < 0$.

In this paper let us consider another condition on the distribution $T_0$ defined by

\[(II) \quad g\left( (A\phi - \phi A)X, Y \right) = 0 \]

for any $X$ and $Y$ in $T_0$, which is a weaker condition than the structure tensor $\phi$ and the second fundamental tensor $A$ commute with each other.

On the other hand, Okumura [15] and Montiel and Romero [14] have considered real hypersurfaces of $P_n(C)$ or $H_n(C)$ satisfying $\phi A = A\phi$, and have shown respectively that they are congruent to real hypersurfaces of type $A_1$, $A_2$ in $P_n(C)$ and of type $A_0$, $A_1$ and $A_2$ in $H_n(C)$. So real hypersurfaces of these types naturally satisfy the condition (II). Then under the condition (II) we are able to give a complete classification of real hypersurfaces in a complex space form $M_n(c)$ which have $\eta$-parallel curvature tensor. That is, we have the following

**Theorem.** Let $M$ be a real hypersurface of $M_n(c)$, $c \neq 0$, and $n \geq 3$, with $\eta$-parallel curvature tensor. If it satisfies the condition (II), then $M$ is locally congruent to one of the following spaces:

1. In case $M_n(c) = P_n(C)$
   (A1) a tube of radius $r$ over a hyperplane $P_{n-1}(C)$, where $0 < r < \frac{n}{2}$,
   (A2) a tube of radius $r$ over a totally geodesic $P_k(C)$ ($1 \leq k \leq n - 2$), where $0 < r < \frac{n}{2}$,

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(R) a ruled real hypersurface foliated by totally geodesic complex hypersurface $P_{n-1}(\mathbb{C})$.

(2) In case $M_n(c) = H_n(\mathbb{C})$
   (A_0) a horosphere in $H_n(\mathbb{C})$, i.e., a Montiel tube,
   (A_1) a tube of a totally geodesic hyperplane $H_k(\mathbb{C})$ ($k = 0$ or $n - 1$),
   (A_2) a tube of a totally geodesic $H_k(\mathbb{C})$ ($1 \leq k \leq n - 2$),
   (R) a ruled real hypersurface foliated by totally geodesic complex hypersurface $H_{n-1}(\mathbb{C})$.

Moreover, we note that real hypersurfaces of type $A_1$, $A_2$ and ruled real hypersurfaces in $P_n(\mathbb{C})$ and of type $A_0$, $A_1$, $A_2$ and ruled real hypersurfaces in $H_n(\mathbb{C})$ have $\eta$-parallel second fundamental tensors (see [7], [11], [12], [17] and [18]). So, naturally they have $\eta$-parallel curvature tensor and satisfy the condition (11).

2. Preliminaries

First of all, we recall fundamental properties of real hypersurfaces of a complex space form. Let $M$ be a real hypersurface of a complex $n$-dimensional complex space form $M_n(c)$ of constant holomorphic sectional curvature $c(\neq 0)$ and let $C$ be a unit normal field on a neighborhood of a point $x$ in $M$. We denote by $J$ an almost complex structure of $M_n(c)$. For a local vector field $X$ on a neighborhood of $x$ in $M$, the transformation of $X$ and $C$ under $J$ can be represented as

$$JX = \phi X + \eta(X)C, \quad JC = -\xi,$$

where $\phi$ defines a skew-symmetric transformation on the tangent bundle $TM$ of $M$, while $\eta$ and $\xi$ denote a 1-form and a vector field on a neighborhood of $x$ in $M$, respectively. Moreover, it is seen that $g(\xi, X) = \eta(X)$, where $g$ denotes the induced Riemannian metric on $M$. By properties of the almost complex structure $J$, the set $(\phi, \xi, \eta, g)$ of tensors satisfies

$$\phi^2 = -I + \eta \otimes \xi, \quad \phi \xi = 0, \quad \eta(\phi X) = 0, \quad \eta(\xi) = 1,$$

where $I$ denotes the identity transformation. Accordingly, the set is so called an almost contact metric structure. Furthermore the covariant derivative of the structure tensors are given by

$$\nabla_X \phi Y = \eta(Y)AX - g(AX, Y)\xi, \quad \nabla_X \xi = \phi AX,$$

where $\nabla$ is the Riemannian connection of $g$ and $A$ denotes the shape operator with respect to the unit normal $C$ on $M$.
Since the ambient space is of constant holomorphic sectional curvature $c$, the equation of Gauss and Codazzi are respectively given as

$$R(Y, Z)U = \frac{c}{4} \left\{ g(Z, U)Y - g(Y, U)Z + g(\phi Z, U)\phi Y - g(\phi Y, U)\phi Zight.$$ \[\left. - 2g(\phi Y, Z)\phi U\right\} + g(AZ, U)AY - g(AY, U)AZ,$$

(2.3) \quad (\nabla_X A)Y - (\nabla_Y A)X = \frac{c}{4} \left\{ \eta(X) \phi Y - \eta(Y) \phi X - 2g(\phi X, Y)\xi \right\},$

where $R$ denotes the Riemannian curvature tensor of $M$ and $\nabla_X A$ denotes the covariant derivative of the shape operator $A$ with respect to $X$.

Now let us suppose that the structure vector $\xi$ is a principal vector with principal curvature $\beta$, that is, $A\xi = \beta \xi$. Then, differentiating this, we have

$$\nabla_X A\xi = (X\beta)\xi + \beta \phi AX - A\phi AX,$$

where we have used (2.1). Then it follows

$$g((\nabla_X A)Y, \xi) = (X\beta)\eta(Y) + \beta g(Y, \phi AX) - g(Y, A\phi AX)$$

for any tangent vector fields $X$ and $Y$ on $M$. By the equation of Codazzi (2.3), we have

$$2A\phi AX - \frac{c}{2} \phi X = \beta(\phi A + A\phi)X.$$

### 3. Proof of the main theorem

Let $M$ be a real hypersurface in a complex space form $M_n(c), c \neq 0$, and let $T_0$ be a distribution defined by a subspace $T_\xi(x) = \{ X \in T_x M \mid X \perp \xi(x) \}$ for any point $x$ in $M$. Now let us prove the main theorem in the introduction. For this purpose we recall

**Lemma 3.1** [9]. Let $M$ be a real hypersurface of $M_n(c), c \neq 0$. If $M$ satisfies the condition (II), then we have

$$g((\nabla_X A)Y, Z) = \mathcal{S}g(AX, Y)g(Z, V),$$

where $\mathcal{S}$ denotes the cyclic sum with respect to $X, Y$ and $Z$ in $T_0$ and $V$ stands for the vector field defined by $\nabla_\xi \xi$.
Let $M$ be a real hypersurface of an $n(\geq 3)$-dimensional complex space form $M_n(c)$ with $\eta$-parallel curvature tensor. That is, for any $X$, $Y$, $Z$ and $U$, $V$ orthogonal to $\xi$ we have $g\left( (\nabla_X R)(Y, Z)U, V \right) = 0$, where the covariant derivative of the curvature tensor $R$ is defined by

$$
$$

Moreover, the formula (2.1) gives $g\left( (\nabla_X \phi)Y, Z \right) = 0$ for any $X$, $Y$ and $Z$ orthogonal to $\xi$. By these formulas the covariant derivative of (2.2) implies

$$
g\left( (\nabla_X A)Z, U \right) g(AY, V) + g(AZ, U)g\left( (\nabla_X A)Y, V \right) - g\left( (\nabla_X A)Y, U \right) g(AZ, V) - g(AY, U)g\left( (\nabla_X A)Z, V \right) = 0.
$$

Then substituting the formulas (3.1) and (3.2) implies

$$
\Theta \sum_{X, Y, U} \left\{ g(AX, Z)g(U, W) \right\} g(AY, V) \\
+ g(AZ, U) \Theta \sum_{X, Y, V} \left\{ g(AX, Y)g(V, W) \right\} \\
- \Theta \sum_{X, Y, U} \left\{ g(AX, Y)g(U, W) \right\} g(AZ, V) \\
- g(AY, U) \Theta \sum_{X, Z, V} \left\{ g(AX, Z)g(V, W) \right\} = 0,
$$

where $\Theta \sum_{X, Y, Z}$ denotes the cyclic sum of $X, Y$ and $Z \in T_0$. So let us write (3.3) in such a way that

$$
\left\{ g(AX, Z)g(U, W) + g(AZ, U)g(X, W) + g(AU, X)g(Z, W) \right\} g(AY, V) \\
+ \left\{ g(AX, Y)g(V, W) + g(AY, V)g(X, W) + g(AV, X)g(Y, W) \right\} g(AZ, U) \\
- \left\{ g(AX, Y)g(U, W) + g(AY, U)g(X, W) + g(AU, X)g(Y, W) \right\} g(AZ, V) \\
- \left\{ g(AX, Z)g(V, W) + g(AZ, V)g(X, W) \\
+ g(AV, X)g(Z, W) \right\} g(AY, U) = 0,
$$

where $W = \phi A\xi$ stands for the vector field defined by $\nabla g\xi$. 

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(3.4) can be rearranged in such a way that
\[
\begin{align*}
&\{ g(AX, Z)g(AY, V) - g(AX, Y)g(AZ, V) \} g(U, W) \\
&+ \{ g(AZ, Y)g(AZ, U) - g(AX, Z)g(AY, U) \} g(V, W) \\
&+ 2\{ g(AZ, U)g(AY, V) - g(AY, U)g(AZ, V) \} g(X, W) \\
&+ \{ g(AU, X)g(AY, Y) - g(AV, X)g(AU, Y) \} g(Z, W) \\
&+ \{ g(AV, X)g(AZ, U) - g(AU, X)g(AZ, V) \} g(Y, W) = 0
\end{align*}
\]
for any \( X, Y, Z \) and \( U, V \in T_0 \).

We have assumed that the structure vector field \( \xi \) is not principal. Then there exists an open subset \( \mathcal{U} \) of this point, on which we can define a unit vector field \( U_0 \) orthogonal to \( \xi \) in such a way that
\[
\beta U_0 = A\xi - g(A\xi, \xi)\xi = A\xi - \alpha\xi,
\]
where \( \beta \) denotes the length of vector field \( A\xi - \alpha\xi \) and \( \beta(p) \neq 0 \) for any point \( p \) in \( \mathcal{U} \). This implies the vector \( W(p) \) is non-zero at any point \( p \) in \( \mathcal{U} \), where \( \mathcal{U} \) denotes the set \( \mathcal{U} = \{ p \in M \mid \beta(p) \neq 0 \} \) in \( M \). Then \( W \) becomes \( W = \beta \phi U_0 \).

Let us continue our discussion on the open set \( \mathcal{U} \). By putting \( U = U_0 \) and \( Y = U_0 \) in (3.5) and using the fact \( g(U_0, W) = \beta g(U_0, \phi U_0) = 0 \), we have
\[
\begin{align*}
&\{ g(AX, U_0)g(AZ, U_0) - g(AU_0, U_0)g(AX, Z) \} g(V, W) \\
&+ 2\{ g(AZ, U_0)g(AU_0, V) - g(AU_0, U_0)g(AZ, V) \} g(X, W) \\
&+ \{ g(AU_0, X)g(AU_0, V) - g(AV, X)g(AU_0, U_0) \} g(Z, W) = 0.
\end{align*}
\]
From this, putting \( Z = V \), it follows
\[
\begin{align*}
&\{ g(AX, U_0)g(AV, U_0) - g(AU_0, U_0)g(AX, V) \} g(V, W) \\
&+ \{ g(AV, U_0)g(AU_0, V) - g(AU_0, U_0)g(AV, V) \} g(X, W) = 0.
\end{align*}
\]
From this, also putting \( V = W = \beta \phi U_0 \), we have
\[
\begin{align*}
&\beta\{ g(AX, U_0)g(A\phi U_0, U_0) - g(AU_0, U_0)g(A\phi U_0, X) \} \|W\|^2 \\
&+ \beta^2\{ g(A\phi U_0, U_0)g(AU_0, \phi U_0) - g(AU_0, U_0)g(A\phi U_0, \phi U_0) \} g(X, W) = 0.
\end{align*}
\]
So, if we take $X = W = \beta \phi U_0$ in (3.8), we have on $\mathcal{U}$

$$\beta^2 \left\{ g(A\phi U_0, U_0)g(A\phi U_0, U_0) - g(AU_0, U_0)g(A\phi U_0, \phi U_0) \right\} \|W\|^2 = 0.$$  

From this, together with (3.8), it follows

$$g(A\phi U_0, U_0)AU_0 = g(AU_0, U_0)A\phi U_0 + \gamma \xi. \quad (3.9)$$

On the other hand, by the condition (II) we know

$$g(A\phi U_0, U_0) = 0, \quad (3.10)$$

because

$$g(A\phi U_0, U_0) = g(\phi AU_0, U_0) = -g(AU_0, \phi U_0) = -g(U_0, A\phi U_0).$$

So (3.9),(3.10) and the condition (II) imply

$$g(AU_0, U_0) = 0. \quad (3.11)$$

In fact, if we suppose $g(AU_0, U_0) \neq 0$, then (3.9) and (3.10) imply $A\phi U_0 = 0$. From this, together with the condition (II) it follows

$$0 = g(A\phi U_0, \phi U_0) = g(AU_0, U_0),$$

a contradiction.

By virtue of (3.10) and (3.11), the formula (3.6) reduces to

$$g(AX, U_0)g(AZ, U_0)g(V, W) + 2g(AX, U_0)g(AU_0, V)g(X, W) + g(AU_0, X)g(AU_0, V)g(Z, W) = 0 \quad (3.12)$$

for any $X$, $Z$, and $V$ in $T_0$. From this, putting $X = \phi U_0$ and using (3.10), we have

$$g(AX, U_0) = 0 \quad (3.13)$$

for any $X \in T_0$. This means $AX = 0$ for any $X$ orthogonal to $\xi$ and $U_0$. For this, let us replace $U$ in (3.5) by $\phi U_0$ and use (3.13) and the condition (II) to this obtained equation. Then it follows that for any $X$, $Y$, $Z$ and $V$ in $T_0$

$$g(AX, Z)g(AY, V) - g(AX, Y)g(AZ, V) = 0. \quad (3.14)$$

By contracting (3.14) with respect to $Y$ and $V$, we have the following.

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Lemma 3.2. $A^2 - (h - \alpha)A - \beta^2 U_0 \otimes U_0 = 0$ on $T_0$, where $h$ denotes the trace of the second fundamental tensor $A$ of $M$.

Also the formula (3.14) gives for any $X, Y$ and $Z$ in $T_0$ orthogonal to $U_0$

\[(3.15)\quad g(AX, Y)AZ = g(AY, Z)AX = g(AZ, X)AY.\]

Let us denote by $T'_0$ a distribution defined by a subspace

\[T'_0(x) = \{X \in T_xM \mid X \perp \xi_x, U_0x\}\]

in $T_xM$ at any $x \in M$. Then the distribution $T'_0$ is the orthogonal complement of the subspace $[\xi, U_0]$ spanned by two vectors $\xi$ and $U_0$. From this, we consider the following two cases such that $g(AX, Y) = 0$ for any $X, Y \in T'_0$ or $g(AX, Y) \neq 0$ for some $X, Y \in T'_0$.

For the first case where $g(AX, Y) = 0$ for any $X, Y \in T'_0$, we have the conclusion

\[(3.16)\quad AX = 0\]

for any $X \perp \xi, U$, because $g(AX, \xi) = 0$ and $g(AX, U_0) = 0$, by (3.13).

Now let us consider the next case such that $g(AX_0, Y_0) \neq 0$ for some $X_0, Y_0$ contained in the distribution $T'_0$. Then in such a case (3.15) implies

\[(3.17)\quad AZ = \frac{g(AY_0, Z)}{g(AX_0, Y_0)}AX_0 = \frac{g(AX_0, Z)}{g(AX_0, Y_0)}AY_0\]

for any $Z \in T'_0$. In this case let us denote by $\tilde{X}_0$ the non-vanishing unit-vector defined by

\[(3.18)\quad \tilde{X}_0 = \frac{AX_0}{\|AX_0\|}\]

for any $X_0 \in T'_0$. Then the vector $\tilde{X}_0$ is contained in the distribution $T'_0$, because

\[A\xi = \alpha\xi + \beta U_0, \quad AU_0 = \beta \xi.\]

Accordingly, by (3.17) and (3.18), we have the following for any $Z \perp \xi, U_0, \tilde{X}_0$ and for some $X_0, Y_0 \in T'_0$

\[(3.19)\quad AZ = \frac{g(AY_0, Z)}{g(AX_0, Y_0)}AX_0 = \frac{g(AX_0, Z)}{g(AX_0, Y_0)}AY_0 = 0,\]
and

\[(3.20) \quad A\tilde{X}_0 = \frac{g(AY_0,AX_0)}{g(AX_0,Y_0)} AX_0 = \frac{g(A^2X_0,Y_0)}{g(AX_0,Y_0)} AX_0. \]

On the other hand, Lemma 3.2 gives for some \(X_0, Y_0 \in T'_0\)

\[g(A^2X_0,Y_0) - (h - \alpha)g(AX_0,Y_0) = 0.\]

From this and together with (3.20), it follows

\[(3.21) \quad A\tilde{X}_0 = (h - \alpha)\tilde{X}_0.\]

On the other hand, by (3.13) and the condition (II), it can be easily seen that \(\phi U_0\) is orthogonal to \(\xi, U_0\) and \(\tilde{X}_0\). So if we put \(Z = \phi U_0\) in (3.19), we know \(A\phi U_0 = 0\).

From these formulas we know that the expression of the second fundamental tensor \(A\) of \(M\) in \(M_n(c)\) satisfying the conditions (I) and (II) is given by

\[A = \begin{bmatrix} \alpha & \beta & 0 \\ \beta & 0 & h - \alpha \\ 0 & h - \alpha & 0 \end{bmatrix}.\]

Moreover, by the condition (II) we can assert \(h = \alpha\). Indeed, first by (3.13) and the condition (II) we assert

\[g(\phi\tilde{X}_0, U_0) = g(\phi AX_0, U_0)/\|AX_0\| = 0,\]

\[g(\phi\tilde{X}_0, \tilde{X}_0) = g(\phi\tilde{X}_0, \xi) = 0\]

for some \(X_0 \in [\xi,U]^\perp\). That is, the vector \(\phi\tilde{X}_0\) is orthogonal to \(\xi, U_0\) and \(\tilde{X}_0\) for some \(X_0 \in [\xi,U]^\perp\). Then by (3.19), we know \(A\phi \tilde{X}_0 = 0\). Also (3.21) implies \(\phi AX_0 = (h - \alpha)\phi \tilde{X}_0\). From these formulas, by putting \(X = X_0\) and \(Y = \phi X_0\) into the condition (II), we have

\[0 = g((A\phi - \phi A)\tilde{X}_0, \phi\tilde{X}_0) = g(\phi AX_0, \phi\tilde{X}_0) = -(h - \alpha)\|\phi\tilde{X}_0\|^2.\]

Accordingly, we get the above assertion.
So we conclude that rank $A \leq 2$, that is,

$$A\xi = \alpha\xi + \beta U_0, \quad AU_0 = \beta \xi \quad \text{and} \quad AX = 0$$

for any $X \perp \xi, U_0$. This means $g(AX,Y) = 0$ for any vector field $X,Y \in T_0$ on $\mathcal{U} = \{ p \in M \mid \beta(p) \neq 0 \}$. Thus $\mathcal{U}$ is congruent to an open part of ruled real hypersurfaces.

Now let us suppose that Int $(M - \mathcal{U})$ is not empty. On this subset the function $\beta$ vanishes identically and the structure vector field $\xi$ is principal. It is seen in [8] and [12] that the corresponding principal curvature $\alpha$ is constant on the interior of $M - \mathcal{U}$, because this is a local property. Then by the condition (1.1) and the fact that $\xi$ is principal we have $A\phi - \phi A = 0$ on Int $(M - \mathcal{U})$. For any principal vector $X$ in $T_0$ with corresponding principal curvature $\lambda$, we have

$$2(\lambda - \alpha)A\phi X = \left(\frac{\alpha}{2} + \alpha \lambda\right) \phi X$$

by (2.6). Using the above two equations we get

$$2\lambda^2 - 2\alpha \lambda - \frac{\alpha}{2} = 0,$$

from which it follows that all principal curvatures are non-zero constant on the interior of $M - \mathcal{U}$. This means that all principal curvatures except $\alpha$ are non-zero constants on $M - \mathcal{U}$. By the continuity of principal curvatures, $M - \mathcal{U}$ is $M$ itself and the subset $\mathcal{U}$ is empty. That is, $\xi$ is principal on $M$.

Then by theorems of Okumura [15] for $c > 0$ and Montiel and Romero [14] for $c < 0$, we conclude that $M$ is congruent to real hypersurfaces of type $A_1$, $A_2$ for $c > 0$ and of type $A_0 - A_1$ and $A_2$ for $c < 0$ respectively.

When we suppose that the set Int $(M - \mathcal{U})$ is empty, the open set $\mathcal{U}$ becomes a dense subset of $M$. By the continuity of principal curvatures again we see that the shape operator satisfies the condition (3.22) on the whole $M$. Accordingly we get $g(\nabla X\xi, \xi) = -g(\nabla X\xi, Y) = -g(\phi AX, Y) = 0$ by (2.1), which means that $\nabla X\xi - \nabla Y\xi$ is also contained in $T_0$. Hence the distribution $T_0$ is integrable on $M$. Moreover the integral manifold of $T_0$ can be regarded as the submanifold of codimension 2 in $M_n(c)$ whose normal vectors are $\xi$ and $C$. Since we have $\bar{g}(\nabla X\xi, \xi) = g(\nabla X\xi, \xi) = 0$ and $\bar{g}(\nabla X\xi, C) = -\bar{g}(\nabla X\xi, Y) = g(AX, Y) = 0$ for any vector fields $X$ and $Y$ in $T_0$ by (3.22), where $\nabla$ denotes the Riemannian connection of $M_n(c)$, it is seen that the submanifold is totally geodesic in $M_n(c)$. Since $T_0$ is $J$-invariant, its integral manifold is a complex submanifold and therefore it is a complex space form $M_{n-1}(c)$. Thus $M$ is locally congruent to a ruled real hypersurface. This completes the proof of our Theorem. □
Remark 3.1. When a real hypersurface $M$ in $M_n(c)$ is locally congruent to a real hypersurface of type $B$, we know that by a theorem of Kimura and Maeda [11] for $c > 0$ and Suh [17] for $c < 0$ respectively the second fundamental tensor of $M$ is $\eta$-parallel. So it naturally satisfies the condition (I), that is, $\eta$-parallel curvature tensor. But its form can not satisfy the condition (II).

References


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