Flavour changing neutral currents in intersecting brane models

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Abstract: Intersecting D-brane models provide an attractive explanation of family replication in the context of string theory. We show, however, that the localization of fermion families at different brane intersections in the extra dimensions introduces flavour changing neutral currents mediated by the Kaluza-Klein excitations of the gauge fields. This is a generic feature in these models, and it implies stringent bounds on the mass of the lightest Kaluza-Klein modes (becoming severe when the compactification radii are larger than the string length). We present the full string calculation of four-fermion interactions in models with intersecting D-branes, recovering the field theory result. This reveals other stringy sources of flavour violation, which give bounds that are complementary to the KK bounds (i.e. they become severe when the compactification radii are comparable to the string length). Taken together these bounds imply that the string scale is larger than $M_s \gtrsim 10^2 \text{TeV}$, implying that non-supersymmetric cases are phenomenologically disfavoured.

Keywords: Field Theories in Higher Dimensions, D-branes, Compactification and String Models.
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1. Introduction

String theory is, to date, the only known candidate for a consistent description of gauge and gravitational interactions. However, there is a large multiplicity of possible vacua and, in general, explicit calculations are difficult to perform. Consequently it is unlikely that we will be able to identify the “correct” string vacuum from first principles, in order to confront string theory directly with observed physics. As an alternative, we can scan different models and single out the cases that reproduce at low energies the main features of the Standard Model (SM) such as non-abelian gauge interactions, three families of chiral fermions, hierarchical Yukawa interactions, and so on. In particular, Calabi-Yau or orbifold compactifications of the heterotic string have been extensively studied. The realisation of D-branes as dynamical objects of string theory has opened new avenues in the search for phenomenologically viable string models. The elusive property of chirality in the SM can for example be accomplished in open string models either by locating D-branes at singularities or by allowing D-branes to intersect at non-trivial angles. The latter possibility, which is the subject of this paper, has proven very promising and a great deal of effort has been devoted to the study of viable models and their phenomenology.

In this paper we want to address an important phenomenological issue in models of intersecting D-branes that has not yet been considered. Branes wrapping compact cycles intersecting at non-trivial angles give rise to several copies of chiral fermions living at the intersections. This multiplicity is generally thought to be an attractive feature of these models, since it leads to a nice explanation for family replication. However, the different
intersection points are localized at separate points in the target space, leading to fermion non-universal couplings to the gauge boson Kaluza-Klein (KK) excitations thereby inducing Flavour Changing Neutral Currents (FCNCs) in the physical basis. (This property was first pointed out in the context of brane world models in ref. [18].) The appearance of FCNCs, which is a quite generic feature of models with intersecting D-branes, is particularly relevant here because most of the favoured models have to date been non-supersymmetric. (Models with intersecting D-branes may have a non-minimal Higgs structure which can also lead to the presence of FCNC. This issue is however very model dependent and can be absent in particular models. Thus we will not pursue its study further here.) The stringent experimental bounds on these couplings compromise the necessarily low scale of non-supersymmetric models, so that supersymmetric set-ups (which are unfortunately rather hard to find) are greatly preferred.

In the next section 2, we review the generation of FCNCs induced by the massive KK modes of bulk gauge bosons when fermions are localized at displaced points in periodic extra dimensions. The case of orbifold compactification was studied in detail in the second reference of [18] (see also ref. [19]). In section 3 we describe the main features of models with intersecting D-branes and study in a quantitative way the amount of FCNCs induced by the KK excitations of gauge bosons. We also show how this field theoretic calculation can be derived in a full string calculation of flavour-violating four fermion interactions. In section 4, we discuss other stringy sources of flavour violation beyond the KK one. Taken together, the resulting bounds provide a strong constraint on the string scale, $M_s \gtrsim 10^2$ TeV.

2. Flavour changing neutral currents from gauge boson Kaluza-Klein modes

To illustrate how FCNCs appear in these models let us consider a U(1) gauge field (the generalisation to non-abelian groups is straight-forward and does not modify qualitatively the main results regarding FCNC) living in a 5 dimensional space, where the extra dimension $y$ is compact and of length $L$ (i.e., the field lives in the world-volume of a D4-brane wrapping a 1-cycle on a torus). Let us also suppose that fermions are four-dimensional fields $f_i$, with the different families localized at different points in the extra dimension $y = y_i$. The relevant part of the lagrangian can be written as (a sum over $i$ understood)

$$\mathcal{L}_5 = -\frac{1}{4} F_{MN} F^{MN} + i \bar{f}_i \gamma^\mu D_\mu f_i \delta(y - y_i),$$

where $F_{MN} = \partial_M A_N - \partial_N A_M$, $D_\mu = \partial_\mu + i g A_\mu$ and $M, N$ run over all space-time dimensions while $\mu, \nu = 0, 1, 2, 3$. The dependence on the extra dimension of the gauge fields $A_\mu$ can be expanded as

$$A_\mu(x, y) = \frac{1}{\sqrt{L}} A_\mu^{(0)}(x) + \sqrt{\frac{2}{L}} \sum_{n=1}^{\infty} \left[ \cos \frac{2\pi n y}{L} A_\mu^{(n)}(x) + \sin \frac{2\pi n y}{L} A_\mu^{(n)}(x) \right] .$$
Integrating the action over $y$ we obtain a 4-dimensional theory (in the unitary gauge$^1$):

$$\mathcal{L} = -\frac{1}{4} \left[ F_{\mu\nu}^{(0)2} + \sum_{n=1}^{\infty} \left( F_{\mu\nu}^{(n)2} + F_{\nu\mu}^{(n)2} \right) \right] + \frac{1}{2} \sum_{n=1}^{\infty} \left( \frac{2\pi n}{L} \right)^2 \left( A_{\mu}^{(n)2} + A_{\mu}^{(n)2} \right) +$$

$$+ i \bar{f}_i \gamma^\mu \left[ \partial_\mu + igA_\mu^{(0)} + ig\sqrt{2} \sum_{n=1}^{\infty} \left( A_{\mu}^{(n)} \cos \frac{2\pi ny_i}{L} + A_{\mu}^{(n)} \sin \frac{2\pi ny_i}{L} \right) \right] f_i,$$  \hspace{1cm} (2.1)

where we have defined $g \equiv g_5/\sqrt{L}$. Notice that there are two KK excitations, $A_\mu^{(n)}$ and $A_{\mu}^{(n)}$, at each mass level $M_n = 2\pi n/L$ and that the couplings of the different fermions to these excitations depend on the position of the fermion in the extra dimension: they are $g_i^n = \sqrt{2} g \cos(M_n y_i)$ and $g_i^n = \sqrt{2} g \sin(M_n y_i)$, respectively. These flavour-dependent couplings will generate FCNCs in the basis of mass eigenstates $^{[15]}$. The coupling of the two KK modes at the $n$–th level to the fermions can be written in terms of currents as

$$\mathcal{L}_n = A_{\mu}^{(n)} J^{(n)\mu} + A_{\mu}^{(n)} J^{(n)\mu}.$$

Under a unitary transformation $f_i = U_{a\bar{a}} f_{\bar{a}}$ from current eigenstates ($f_\bar{a}$) to mass eigenstates ($f_a$), $J^{(n)\mu}$ becomes

$$J^{(n)\mu} = \bar{f}_a \gamma^\mu U_{a\bar{a}} g_{\bar{a}b} U_{\bar{b}} f_b,$$

with a similar expression for $J^{(n)\mu}$. If $g^n_i \neq g^n_j$ the product of unitary matrices does not cancel and FCNCs are generated. We can then integrate out the heavy KK modes and obtain (at first order in $M_n^2$) flavour violating four-fermion contact interactions

$$\mathcal{L}_4 = -\frac{1}{2} \sum_n J^{(n)\mu} J_{\mu}^{(n)} + J^{(n)\mu} J_{\mu}^{(n)} M_n^2. \hspace{1cm} (2.2)$$

Note that the four fermion amplitudes will be a sum of contributions proportional to

$$\sum_{n=1}^{\infty} g_i^n g_j^n + g_i^n g_j^n = 2g^2 \sum_{n=1}^{\infty} \cos[M_n(y_i - y_j)] M_n^2$$

$$= \left( \frac{gL}{2\pi} \right)^2 \left[ \text{Li}_2(e^{2\pi i(y_i - y_j)/L}) + \text{Li}_2(e^{-2\pi i(y_i - y_j)/L}) \right], \hspace{1cm} (2.3)$$

where $\text{Li}_n(z) = \sum_{k=1}^{\infty} z^k/k^n$ is the polylogarithm function and we have used the explicit form of the KK masses in the second line. The presence of the second tower of KK modes is necessary to preserve the global translation invariance of the circle in such a way that only the relative distances between fermions are observable. The extension to $D > 1$ extra dimensions can be trivially obtained from the expressions above (except for the last equality in eq. (2.3)), by changing the coordinate $y \rightarrow \vec{y} \equiv (y_1, \ldots, y_D)$, the length factor to the volume factor $\sqrt{L} \rightarrow \sqrt{V}$, the KK indices $n \rightarrow \vec{n} = (n_1, \ldots, n_D)$, the masses

$^1$We do not write the KK expansion of the component of the gauge boson along the extra dimension since it is not relevant for our calculation. The massive modes are the Goldstone bosons associated to the five-dimensional gauge invariance broken by the compactification (they decouple in the unitary gauge), whereas the zero mode couples universally and does not generate any FCNCs.
$M_n \rightarrow M_{\tilde{h}} = 2\pi(n_1/L_1, \ldots, n_D/L_D)$ and by extending the sum to one over a hemisphere on the $D$–dimensional lattice. For higher dimensional branes the sum over KK modes diverges and requires a UV cut-off (radiative corrections could also act as cut-off [20]). The full string calculation, to be presented in section 3.2, provides a natural cut-off in terms of the string scale, which we will adopt in the field theory calculation in next section.

To be more definite, let us calculate the $\Delta S = 2$ operators involved in flavour changing and CP violation in the Kaon system, since they are expected to give the strongest constraint on the compactification scale $L$ [13]. The relevant fermionic currents coupling to the KK excitations of the gluon $G_\mu^A$ are, for a general number of extra dimensions,

$$J_\mu^{A(\alpha)} = (U_{dL}^\dagger)_{ai} g_{\bar{a}}^i (U_{dL})_{is} d_L^\dagger T^{A}_\alpha \gamma_\mu s_L^\beta + \text{L} \rightarrow \text{R} + \text{h.c.},$$

(2.4)

where $g_{\bar{a}}^i = \sqrt{2} g_\beta \cos(M_{\tilde{h}} \cdot \tilde{y}_i)$, $\alpha, \beta$ are colour indices and $T^{A}_\alpha$ are the fundamental representation matrices of SU(3). A similar expression holds for $J_\mu^{A(\bar{a})}$. The resulting four-fermion lagrangian reads

$$-L^{\Delta S=2} = \sum_{\bar{a}} \left\{ \frac{c^{(\bar{a})}_{LL}}{M_n^2} \left( \bar{d}_L^\gamma \epsilon^\mu s_L^\alpha \right) \left( \bar{d}_L^\gamma \epsilon^\mu s_L^\beta \right) + \frac{c^{(\bar{a})}_{RR}}{M_n^2} \left( \bar{d}_R^\gamma \epsilon^\mu s_R^\alpha \right) \left( \bar{d}_R^\gamma \epsilon^\mu s_R^\beta \right) + \frac{c^{(\bar{a})}_{LR}}{M_n^2} \left( \bar{d}_L^\gamma \epsilon^\mu s_R^\beta \right) \left( \bar{d}_R^\gamma \epsilon^\mu s_L^\alpha \right) + \frac{c^{(\bar{a})}_{RL}}{M_n^2} \left( \bar{d}_R^\gamma \epsilon^\mu s_L^\alpha \right) \left( \bar{d}_L^\gamma \epsilon^\mu s_R^\beta \right) + \text{h.c.} \right\},$$

(2.5)

where prime in the sum means that it has to be extended over a hemisphere of the $D$–dimensional lattice. The coefficients are

$$c^{(\bar{a})}_{LL} = \frac{g_\beta^2}{3} \delta^{iL} M_n^2 \sum_{ij} \left( U_{dL}^\dagger \right)_{ai} (U_{dL})_{is} \left( U_{dL}^\dagger \right)_{dj} (U_{dL})_{js} \cos \left[ \tilde{M}_{\tilde{h}} \cdot \left( \tilde{y}_i^L - \tilde{y}_j^L \right) \right],$$

(2.6)

(similarly for $c^{(\bar{a})}_{RR}$ with $L \rightarrow R$) and

$$c^{(\bar{a})}_{LR} = -3c^{(\bar{a})}_{RL} = -2g_\beta^2 \delta^{iL} M_n^2 \sum_{ij} \left( U_{dL}^\dagger \right)_{ai} (U_{dL})_{is} \left( U_{dR}^\dagger \right)_{dj} (U_{dR})_{js} \cos \left[ \tilde{M}_{\tilde{h}} \cdot \left( \tilde{y}_i^L - \tilde{y}_j^L \right) \right].$$

(2.7)

We have included a suppression of the gauge coupling, $g_\beta^2 \rightarrow g_\beta^2 \delta^{-iL} M_n^2$ with $\delta \gtrsim 1$ and $l_s$ the string length, acting as a cut-off at the string scale. This type of suppression will be derived in the string calculation in section 3.2. $U_{dL,R}$ are the unitary matrices diagonalising the down mass matrix

$$\left( U_{dL}^\dagger \right)_{ai} M_{ij}^{dL} (U_{dR})_{jb} = m_{ab}^d \delta_{ab},$$

and we have used $T^{A}_\alpha T^{A}_\gamma = -\delta_{\alpha\beta} \delta_{\gamma\delta}/6 + \delta_{\alpha\delta} \delta_{\beta\gamma}/2$ and the Fierz rearrangements (for anticommuting fields) $(\bar{a}_L \gamma^\mu b_L) (\bar{c}_L \gamma^\mu d_L) = (\bar{a}_L \gamma^\mu d_L) (\bar{c}_L \gamma^\mu b_L)$ and $(\bar{a}_L \gamma^\mu b_L) (\bar{c}_R \gamma^\mu d_R) = -2(\bar{a}_L d_R) (\bar{c}_R b_L)$. In the next section we apply these results to the calculation of FCNCs in explicit models with intersecting D-branes.
3. Intersecting D-brane models

Models with branes intersecting at angles have received a great deal of attention in recent years. In particular, it has been shown that they allow the construction of models containing just the SM spectrum and symmetries in the low energy effective theory. In these models each stack of $N$ D$p$-branes defines a $(p + 1)$-dimensional gauge theory with $U(N) \sim SU(N) \times U(1)$ symmetry. The extra $U(1)$s can become massive by combining with RR-fields, giving rise to unbroken global symmetries at the perturbative level. See also [14] for a recent study of the phenomenology of these extra $U(1)$ gauge bosons. Massless fermions live in the four-dimensional intersections of two stacks of branes, and transform under the bifundamental representation of the corresponding groups. Chirality can be automatic, as it is in the case of D6-branes wrapping factorizable 3-cycles in a six-dimensional torus $T^2 \times T^2 \times T^2$, or obtained by locating the intersections at orbifold fixed points, as is the case with D$(3+n)$-branes on $n$-cycles in $T^{2n} \times R^{6-2n} / \mathbb{Z}_N$, with $n = 1, 2$ [1]. Light scalars live near the intersections with (possibly tachyonic) masses depending on the particular values of the angles between branes, allowing for (quasi-)supersymmetric configurations for particular values of the angles [5, 6, 7], and also giving rise to fields with the quantum numbers of the Higgs boson. The multiple wrapping of the branes around compact cycles leads to a number of intersections which explains family replication. As a consequence, in this kind of models different families are necessarily localized at different space-time points. This in turn induces non-universal couplings to the KK excitations of the gauge bosons and FCNCs through the mechanism described in the previous section. The appearance of FCNCs is a quite generic feature of models with intersecting branes. Bearing in mind that similar features occur in more general Calabi-Yau compactifications [2], we will focus here on toroidal compactifications to make the calculations tractable.

3.1 Field theory calculation

In order to quantify the amount of flavour violation present in this class of models, we will concentrate on a particular case that can be considered as a good starting point for a fully realistic model of intersecting D-branes. It consists of an orientifold compactification of Type IIA string theory on a 6-torus $T^2 \times T^2 \times T^2$ with four stacks of D6-branes, called baryonic (a), left (b), right (c) and leptonic (d), giving rise to the gauge groups $SU(3), SU(2)_L, U(1)_R$ and $U(1)_{\text{lepton}}$, respectively. The orientifold projection is implemented as $\Omega \mathcal{R}$, where $\Omega$ is the world-sheet parity and $\mathcal{R}$ is a reflection with respect to the first component of each 2-torus, $\mathcal{R}Z_I = Z_I$, with $Z_I = X_{2I+2} + iX_{2I+3}$, $I = 1, 2, 3$. The D6-branes have 4 extended space-time plus three compact dimensions, each of which wraps a 1-cycle on each of the three 2-tori. Let us denote by $(n^I_a, m^I_a)$ the 1-cycle that the $a$ stack of branes wraps, going $n^I_a$ times around the real dimension and $m^I_a$ times around the imaginary dimension of the complex $I$-th torus. Each brane $a$ is accompanied by an orientifold image $a^*$ with wrapping numbers $(n_a^I, -m_a^I)$. Chiral fermions live in the four-dimensional intersections between the different branes, transforming in the bifundamental representation of $SU(N)$.

Another possibility, phenomenologically very appealing, is the appearance of orthogonal or symplectic groups in the presence of orientifold fixed planes.
Figure 1: Brane configuration in a model of D6-branes intersecting at angles. The leptonic sector is not represented while the baryonic, left, right and orientifold image of the right are respectively the dark solid, faint solid, dashed and dotted. The intersections corresponding to the quark doublets ($i = -1, 0, 1$), up type singlets ($j = -1, 0, 1$) and down type singlets ($j^* = -1, 0, 1$) are denoted by an empty circle, full circle and a cross, respectively. All distance parameters are measured in units of $2\pi R$ with $R$ the corresponding radius (except $\epsilon^{(3)}$ which is measured in units of $6\pi R$).

representation of the corresponding groups, $(N_a, N_b)$ for an $a, b$ intersection and $(N_a, N_b^*)$ for an $a, b^*$ intersection. Their number depends on a purely topological property, the net intersection number

$$I_{ab} = (n_1^1 m_1^1 - m_1^1 n_1^1) \times (n_2^2 m_2^2 - m_2^2 n_2^2) \times (n_3^3 m_3^3 - m_3^3 n_3^3),$$

with negative intersection numbers corresponding to a positive number of opposite chirality fermions. Consistency conditions given by RR tadpole cancellation plus the requirement of a realistic massless spectrum impose stringent restrictions in the possible configurations of intersecting D-branes, becoming even stronger if supersymmetry is to be preserved. Although some progress has been made in this direction, either with extra exotic states in the spectrum [9, 10], or with locally supersymmetric models [11], in which quadratic corrections only appear at two loop order, the search for fully realistic supersymmetric models has proven an extremely difficult task. A final ingredient of these models relevant for us is the structure of Yukawa couplings between two fermions and the Higgs boson (which in the class of models we are considering arises from the intersection between the left and the right or the orientifold image of the right branes). The main contribution to Yukawa couplings comes from worldsheet instantons, given by the exponential of (minus) the area of the worldsheet stretching between the three intersections involved. Complex structure can appear in the Yukawa couplings when the $B$-field or Wilson lines are turned on. (See ref. [15] for a recent calculation of Yukawa couplings in general toroidal and certain Calabi-Yau compactifications.)

For the sake of concreteness we consider the particular model presented recently in ref. [14]. Although it does not give rise to a realistic pattern of fermion masses and mixing angles, the features that are of relevance for our discussion are much clearer in this model than in more realistic but involved ones. The model is represented graphically in figure 1 where we have omitted the leptonic sector for clarity. The relevant geometry for flavour physics takes place in the second and third tori with no inter-generation distances occurring in the first torus. Quark doublets, which live at the intersections between the baryonic (dark
solid) and the left (faint solid) branes, are labelled in the plot by $i = -1, 0, 1$. Up-type quark singlets ($j = -1, 0, 1$) live at the intersections between the baryonic and the right (dashed) branes while down-type singlets ($j = -1, 0, 1$) live at the intersection between the baryonic and the orbifold image of the right (dotted) brane. Two Higgs fields are localized at the intersection (in the second and third tori) between the left and the right and the image of the right branes respectively. For a particular configuration, namely the ratio of the radii being equal in the second and third tori $R^{(2)}/R^{(3)} = R^{(3)}/R^{(1)}$, the same $\mathcal{N} = 1$ supersymmetry is preserved at all the intersections and the model could in principle be embedded in a bigger $\mathcal{N} = 1$ globally supersymmetric configuration. In particular, the massless particles fill out the spectrum of the MSSM.

The lengths of the cycles that the SU(3) brane wraps on the different tori are, respectively,

$$L_1 = 2\pi R^{(1)}_1, \quad L_I = 2\pi \sqrt{(R^{(I)}_1)^2 + 9(R^{(I)}_2)^2}, \quad I = 2, 3.$$  \hspace{1cm} (3.2)

The relative locations of the different families (distances up to integer multiples of $L_I$) are straight-forward to compute (see table 1) where the meaning of the different coefficients is explained in figure 1 and in the last equation we have written the relative separation for down-type quarks (the up-type case being the same with the substitution $\varepsilon^{(3)} \to -\varepsilon^{(3)}$).

Using the results in the previous section we may write the contribution to the mass difference and CP violation in the Kaon system

$$\Delta m_K = \frac{\text{Re}\langle K^0| - \mathcal{L}^{\Delta S=2}| \bar{K}^0 \rangle}{m_K},$$  \hspace{1cm} (3.3)

and

$$|\epsilon_K| = \frac{\text{Im}\langle K^0| - \mathcal{L}^{\Delta S=2}| \bar{K}^0 \rangle}{2\sqrt{2}m_K \Delta m_K}.$$  \hspace{1cm} (3.4)

In the vacuum insertion approximation, the relevant matrix element can be written as (see for instance [21])

$$\langle K^0| - \mathcal{L}^{\Delta S=2}| \bar{K}^0 \rangle = \sum_{\vec{n}} \left[ \frac{2}{3} \left( c^{(\vec{n})}_{LL} + c^{(\vec{n})}_{RR} \right) + \left( \frac{1}{2} \frac{m_K^2}{(m_s + m_d)^2} + \frac{1}{12} \right) c^{(\vec{n})}_{LR} + \left( \frac{1}{6} \frac{m_K^2}{(m_s + m_d)^2} + \frac{1}{4} \right) c^{(\vec{n})}_{LR} \right] \frac{m_K^2 f_K}{M_{\pi}^2},$$  \hspace{1cm} (3.5)

where $m_{d,s}$ are the masses of the down and strange quarks, respectively, and $f_K = 160\text{ MeV}$.
is the Kaon decay constant. Experimental constraints on the Kaon mass difference and epsilon parameter now place strong bounds on the masses of the KK modes and thus on the length of the cycles that the branes wrap in the different tori. Writing the length of the cycles in terms of the largest one, \( \lambda_I = L_I/L_{\text{max}} \), and requiring the gluon KK contribution to be smaller than the experimental values, \( \Delta m_K = 3.5 \times 10^{-15} \text{ GeV} \) and \( |\epsilon_K| = 2.3 \times 10^{-3} \), we can put a bound on the mass of the lightest gluon KK mode,

\[
M_1 \geq 700 \left( \frac{\sum_n |\text{Re}[\frac{2}{n}(c^{(n)}_{LL} + c^{(n)}_{RR}) + 7.2c^{(n)}_{LR} + 2.6e^{(n)}_{LR}]|}{(n/\lambda)^2} \right) \frac{1}{0.37} \text{ TeV(from } \Delta m_K) ,
\]

and

\[
M_1 \geq 8800 \left( \frac{\sum_n |\text{Im}[\frac{2}{n}(c^{(n)}_{LL} + c^{(n)}_{RR}) + 7.2c^{(n)}_{LR} + 2.6e^{(n)}_{LR}]|}{(n/\lambda)^2} \right) \frac{1}{0.37} \text{ TeV(from } \epsilon_K) ,
\]

where the different coefficients are defined in eqs. (2.6) and (2.7) and we have denoted \( (n/\lambda)^2 \equiv n_1^2/\lambda_1^2 + n_2^2/\lambda_2^2 + n_3^2/\lambda_3^2 \). The bounds have been normalised where the angles are of similar size to those of the CKM matrix, \( \epsilon^{(2)} = \epsilon^{(3)} = 0.2, \epsilon^{(3)} = 0.3, \lambda_I = 1 \) (all cycles of the same size) and to be conservative we have taken only the contribution from the first KK level. One could argue here that no bound is set on the string scale at all, merely the compactification scale. However the string scale should be reasonably close to the compactification scale. It certainly cannot be much smaller if non-negligible Yukawa couplings are to be generated, and it cannot be much larger since the divergent contribution of the KK modes is regulated by a string scale cut-off (as we shall see presently). Furthermore, we will see in the next section that when the string length is of the order of the compactification scale, new stringy sources of flavour violation again banish the string scale to very high values.

Thus, the stringent bounds we have found on the mass of the first KK mode can be translated into strong bounds on the string scale, implying that non-supersymmetric configurations are strongly disfavoured. It should be noted here that the previous calculation should be taken as an estimate of the order of magnitude of generated FCNCs. However, despite the uncertainties, we have been conservative in the actual calculation. Larger mixing angles or a higher cut-off for the multi-dimensional sums can significantly increase the induced FCNCs. For instance, using the string inspired coupling suppression of higher KK modes and taking the string scale to be \( l_s = L_I/20 \) and all the other numbers as above we obtain a bound on the string scale

\[
M_s \gtrsim \begin{cases} 
3200 \text{ TeV}, & \text{from } \Delta m_K, \\
40000 \text{ TeV}, & \text{from } \epsilon_K,
\end{cases}
\]

which corresponds to \( M_1 \gtrsim 1000 \text{ TeV} \) and \( M_1 \gtrsim 12600 \text{ TeV} \), respectively. Of course the bounds obtained depend on many parameters and could be smaller as well in particular models, so we should remark that the expressions above are quite general and can be applied to any model.
3.2 The string theory calculation

In this section we calculate the typical contribution to FCNC processes in models where the chiral matter multiplets come from the intersection of D-branes at angles. We will see that the field theory result is recovered, along with a number of other features. First we shall find a natural stringy explanation for the cut-off which has to be added by hand in the field theory. Second we can consider additional flavour changing processes such as $e^-e^+ \rightarrow \mu^-\tau^+$ that come from the exchange of stretched string modes. From the world-sheet point of view these are additional instanton contributions.

Four fermion interactions have previously been considered for orthogonal D branes in ref. [24]. For theories with branes at angles these processes are particularly important because the sector of chiral fermions is rather independent of the general set up, whereas the scalars are more model dependent and may be tachyonic. The techniques for calculating 3 and 4 point amplitudes with intersecting branes will be presented in detail elsewhere [25]. Here we shall present the 4 point calculation and extract the results necessary for the present analysis, in particular to show that the generation dependence is as described for the field theory.

String states that are stretched between branes at angles are analogous to twisted states in the closed string, and much of the calculation can be made using that technology [23, 22]. Indeed if two D-branes intersect in a single complex dimension with a relative angle $\pi \vartheta$ at the origin, then the complex coordinate $Z(z)$ describing how the world sheet of an open string attached to both branes is embedded has the mode expansion [3]

$$Z = \sqrt{\frac{\alpha'}{2}} \sum_n \frac{\alpha_n + \vartheta - 1}{n + \vartheta - 1} z^{n + \vartheta - 1} + \frac{\tilde{\alpha}_n - \vartheta}{n - \vartheta} z^{n - \vartheta}. \tag{3.9}$$

A similar mode expansion obtains for the fermions with the obvious addition of $1/2$ to the boundary conditions for NS sectors. More generally the massless fermions of interest appear in the Ramond sector with charges, $q_{i=0...3}$ for the 4 complex transverse fermionic degrees of freedom given by one of the following

$$q = \left( \pm \frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2} \right),$$

$$q = \left( \pm \frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2} \right),$$

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depending on the type of intersection. For example D6-branes intersecting in $(T_2)^3$ are of the first kind. The GSO projection leaves only one 4D spinor, and the theory is chiral. For special values of angles (0 for example) supersymmetry may be restored, but generally supersymmetry is completely broken, and the scalars can be heavy or tachyonic. Fermions for D5-branes intersecting in $(T_2)^2 \times C/Z_N$ are of the second kind. Initially the GSO projection will leave only half the space time spinor degrees of freedom leaving a non-chiral theory. Hence a further orbifolding on the 1st complex dimension is required to
get a chiral theory. Finally D4-branes intersecting in $T_2 \times C_2/Z_N$ correspond to the last choice. Again the GSO projection leaves 4 states which need to be further projected out by orbifolding in the $C_2$ dimensions. The particular orbifolding do not effect the Ramond charges above so the quantum part of the amplitude will be unaffected by it. The classical part of the amplitude depends purely on the world sheet areas. However as the orbifolding is orthogonal to the space in which the branes are wrapping, it cannot affect the classical part either. The only effect of the orbifolding is therefore in projecting out the chiralitites above.

The four fermion scattering amplitude is given by a disk diagram with 4 vertex operators $V^{(a)}$ on the boundary. The diagram is then mapped to the upper half plane with vertices on the real axis. The positions of the vertices are fixed by $SL(2, \mathbb{R})$ invariance to $0, x, 1, \infty$ (where $x$ is real) as usual so that the 4 point ordered amplitude is

$$
(2\pi)^4 \delta^4 \left( \sum_a k_a \right) A(1, 2, 3, 4) = \frac{-i}{g_{s R}^2} \int_0^1 dx \langle V^{(1)}(0, k_1)V^{(2)}(x, k_2)V^{(3)}(1, k_3)V^{(4)}(\infty, k_4) \rangle .
$$

(3.10)

To get the total amplitude we have to sum over all possible orderings

$$
A_{\text{total}}(1, 2, 3, 4) = A(1, 2, 3, 4) + A(1, 3, 2, 4) + A(1, 2, 4, 3) + A(4, 3, 2, 1) + A(4, 2, 3, 1) + A(4, 3, 1, 2).
$$

(3.11)

The vertex operators for the fermions are of the form

$$
V^{(a)}(x_a, k_a) = \text{const} \lambda^a u^\alpha S^\alpha \prod_I \sigma^{(I)} e^{-\phi/2} e^{ik_a \cdot Z(x_a)},
$$

(3.12)

where $u^\alpha$ is the space time spinor polarization and $S^\alpha$ is the so called spin-twist operator of the form

$$
S^\alpha = \prod_I : \exp(i q^\alpha_I H_I) : ,
$$

(3.13)

with conformal dimension

$$
h = \sum_I q^2_I.
$$

(3.14)

$\sigma^{(I)}$ is the $\vartheta$ twist operator acting on the $I$–th complex dimension, with conformal dimension

$$
h_I = \frac{1}{2} \vartheta_I (1 - \vartheta_I).
$$

(3.15)

The calculation of the 4-point function of the bosonic twist operators is now analogous to the closed string case [22], and the quantum part follows through with only minor modifications. For simplicity, consider branes intersecting at an angle $\pi \vartheta$ in a sub 2-torus of the compact space. We find a contribution from the $\vartheta$ twisted bosons of

$$
\langle \sigma_+(x_\infty) \sigma_-(1) \sigma_+(x) \sigma_-(0) \rangle = \text{const} \frac{x_\infty x (1 - x)^{-\vartheta(1 - \vartheta)}}{[F(1 - x)F(x)]^{1/2}},
$$

(3.16)
Figure 2: The generic 4 fermion diagram with branes intersecting in a 2-torus.

where $F(x)$ is the hypergeometric function

$$F(x) = F(\theta, 1 - \theta; 1; x) = \frac{1}{\pi} \sin(\theta \pi) \int_0^1 dy y^{-\theta} (1 - y)^{-(1 - \theta)} (1 - xy)^{-\theta}. \quad (3.17)$$

When we collect all the contributions together the dependence on $\theta$ cancels between the bosonic twist fields and the spin-twist fields (the same cancellation in conformal dimension that guarantees massless states in the Ramond sector) giving

$$A(1, 2, 3, 4) = -g_s \alpha' \text{Tr} (\lambda^1 \lambda^2 \lambda^3 \lambda^4 + \lambda^4 \lambda^3 \lambda^2 \lambda^1) \left[ \pi^{(1)} \gamma_{\mu} u^{(2)} \pi^{(4)} \gamma_{\mu} u^{(3)} \right] \times$$

$$\times \int_0^1 dx x^{-1-\alpha' s}(1-x)^{-1-\alpha' t} \frac{1}{[F(1-x)F(x)]^{1/2}} \sum e^{-S_{cl}(x)}, \quad (3.18)$$

where $s = -(k_1 + k_2)^2$, $t = -(k_2 + k_3)^2$, $u = -(k_1 + k_3)^2$ are the usual Mandelstam variables.

The important factor in determining the coupling is then the instanton contribution due to the classical action which is discussed in more detail in ref. [25]. Consider a generic open string four point diagram, as shown in figure 2. In this case the classical action turns out to be

$$S_{cl} = \frac{\sin \theta \pi}{4\pi \alpha' \tau} \left( |v_A'|^2 + \tau^2 |v_B'|^2 \right), \quad (3.19)$$

where

$$v_{A,B}' = \Delta f_{A,B} + nL_{A,B}, \quad (3.20)$$

and we have defined $\tau(x) = \frac{F(1-x)}{F(x)}$. (For $Z_2$ twists, i.e. intersections at right-angles, this would be the modular parameter of a “fake” annulus but it has no such interpretation for more general intersections.) Here $\Delta f_{A,B}$ are the displacements between consecutive vertices along the $A$ and $B$ branes respectively, $n \in \mathbb{Z}$ and $L_{A,B}$ are the vectors in the two torus describing the wrapped D-branes. The leading contribution in this case comes from
strings stretched between $f_1$ and $f_2$ propagating along the $A$-brane (in the $f_3 - f_2$ direction) as shown in figure 2 for which we choose $\Delta f_A = f_3 - f_2$ and $\Delta f_B = f_1 - f_2$. There is an additional contribution to the amplitude, shown in figure 3 where $\Delta f_A = f_1 - f_2$ along the $A$-brane and $\Delta f_B = f_2 - f_3$ along the $B$-brane. This corresponds to diagrams where a string stretched between $f_1$ and $f_2$ along the $A$-brane propagates in the $f_1 - f_2$ direction, i.e. along the $B$-brane, and because of the much larger world sheet area is a subleading contribution for the example shown.

For diagrams where $\Delta f_A$ and $\Delta f_B$ are both non-zero, we expect a world-sheet instanton suppression factor that goes like the world-sheet area. To get this we can use a saddle point approximation for the $x$ integral in $A(1, 2, 3, 4)$ with $\tau(x_s) = |v'_A|/|v'_B|$. The accuracy of this approximation is a function of the width of the saddle, given by $\sqrt{4\pi\alpha'/R_c^2} \sim \frac{1}{R_c}$, where $R_c$ is the compactification scale ($R_c \sim R_A, R_B$). As expected the approximation breaks down when the size of the world sheet is comparable to the D-brane thickness. Substituting back into the action we find

$$A(1, 2, 3, 4) \sim \sum_{\Lambda_A, \Lambda_B} \text{fermion factors} \times \left( \frac{4\pi\alpha'}{R_c} \right)^2 \exp \left( -\frac{1}{2\pi\alpha'} \sin \vartheta \tau |v'_A|/|v'_B| \right). \quad (3.21)$$

We find the expected suppression from the mass of the intermediate stretched string state, times by an instanton suppression given by the area of the world sheet. These expressions will be useful in considering more exotic flavour changing processes such as $ee \rightarrow \tau \mu$.

For the moment however, we are interested in processes that do not explicitly violate flavour, such as $\mu^+\mu^- \rightarrow e^+e^-$. For such processes the intersection separation in the leading term $v'_B = \Delta f_{ee}$ for one pair of twist operators is zero, and so this term cannot be treated as above. Consider the summation over $\Lambda_A$ in $v'_A$ for the other pair, whose

---

**Figure 3:** Subleading contribution to the 4 fermion diagram with branes intersecting in a 2-torus.
separation is \( v_A' = \Delta f_{e\mu} + n L_A \). Poisson resumming we find

\[
\sum e^{-S_{cl}} = \sum_{p_A \in \Lambda_A^*} \sqrt{\frac{4 \pi^2 \alpha' \tau}{L_A^2 \sin \vartheta \pi}} \exp \left[ -\frac{4 \pi^2 \alpha' \tau}{\sin \vartheta \pi} p_A^2 \right] \exp \left[ 2 \pi i \Delta f_{e\mu} \cdot p_A \right] + \text{subleading},
\]

where \( p_A \in \Lambda_A^* \) is summed over the dual lattice

\[
p_A = \frac{n_A}{|L_A|^2} L_A.
\]

This expression describes the leading exchange of gauge bosons plus their KK modes along the \( A \) brane. (The subleading terms (those with \( v'_B = n_B L_B \) with integer \( n_B \neq 0 \) can still be treated using the saddle point approximation above.) To obtain the field theory result, we take the limit of coincident vertices, \( x \to 0 \) or \( x \to 1 \). For example the former contribution gives \( s \)-channel exchanges and can be evaluated using the asymptotics

\[
F(x) \sim 1, \quad \tau \sim F(1-x) \sim \frac{1}{\pi} \sin \vartheta \pi \ln \frac{\delta}{x},
\]

where \( \delta \) is given by the digamma function \( \psi(z) = \Gamma'(z)/\Gamma(z) \)

\[
\delta = \exp \left( 2 \psi(1) - \psi(\vartheta) - \psi(1 - \vartheta) \right).
\]

We find

\[
A(1, 2, 3, 4) = g_s \left[ \gamma^{(1)} \gamma^{(2)} \gamma^{(4)} \right] \frac{\sqrt{4 \pi^2 \alpha'}}{L_A \sqrt{\sin \vartheta \pi}} \times \left( \frac{1}{s} + 2 \sum_{n=1}^{\infty} \frac{\cos (2 \pi \Delta y_{e\mu} p_n)}{s - M_n^2} \right),
\]

where \( M_n = 2 \pi n / L_A \), \( p_n = n L_A / |L_A|^2 \) and we have denoted \( \Delta y_{e\mu} = |\Delta f_{e\mu}| \) to match the field theory calculation, thereby recovering the one-dimensional case we derived in the field theory approximation, provided that \( \alpha' M_n^2 \ll 1 \). That is, the brane separation should again be larger than the brane thickness. Note that we have been a little sloppy in the notation since the indices \( e, \mu \) do not correspond to flavour but to current eigenstates. This expression is thus to be compared with eq. (2.2) before the unitary rotations. The extension to higher dimensional intersecting branes follows straightforwardly, and we now find that the form factor \( \delta - \alpha' M_n^2 \) naturally provides the UV cut-off which in the field theory had to be added by hand. Physically the cut-off arises because the intersection itself has thickness \( \sim \sqrt{\alpha'} \), and thus cannot emit modes with a shorter wavelength.

4. Phenomenological discussion and conclusions

We have seen that a general feature of models with branes intersecting at angles is the appearance of FCNCs mediated by the KK excitations of the gauge bosons living in the world volume of the D-branes. The general four fermion interactions are suppressed by the
square of the compactification scale, with coefficients

\[ c_{\text{KK}} \sim \sum_n \frac{1}{M_n^2} \sim \sum_n \left( \frac{L_c}{2\pi n} \right)^2. \]  

(4.1)

The experimental bounds on flavour changing neutral processes then impose stringent constraints on the compactification scale or, alternatively, on the mass of the lightest gauge boson KK excitation

\[ M_1 \gtrsim 700 \text{ -- } 8800 \text{ TeV}. \]  

(4.2)

This is, however, a conservative bound that arises from a purely field theoretic calculation. Furthermore KK sums require some regularisation for more than one extra dimension that, at this level, has to be put by hand. We obtained a better understanding of the situation by performing a full string calculation of the relevant four fermion interactions. In addition to being an interesting calculation to perform, the string calculation provides us with a natural regularisation of the KK sums and gives us confidence that our physical intuition is correct. An explicit exponential suppression of the coupling in terms of mass of the corresponding KK mode and the string scale is found, leading to the following regularised form of the coefficient of the four fermion interaction

\[ c_{\text{KK}}^{\text{reg}} \sim \sum_n \left( \frac{L_c}{2\pi n} \right)^2 \delta^{-2(2\pi n l_s/L_c)^2}, \]  

(4.3)

with \( \delta \) a number of order one and \( l_s \) the string scale. This dramatically increases the amount of flavour violation (and thus the bound on the compactification scale) in the limit \( l_s \ll L_c \). In the case \( l_s = L_c/20 \) for instance we find,

\[ M_s \gtrsim 3200 \text{ -- } 40000 \text{ TeV}, \]  

(4.4)

corresponding to the mass of the first gauge boson KK mode being

\[ M_1 \gtrsim 1000 \text{ -- } 12600 \text{ TeV}. \]  

(4.5)

On the other hand, if the compactification length is of the same order or even smaller than the string length, then only the KK modes in the first level need to be considered, and the bounds from this source can be much milder.

The string calculation does however give us another source of flavour violation in these models with a different dependence on the compactification and string scales. These correspond to a process with the string stretched along the parallelogram formed by two sets of intersecting D-branes, as shown in figure 4. This can mediate processes such as \( \tau \rightarrow ee\bar{\mu} \).

From the worldsheet point of view these transitions correspond to instanton contributions that are proportional to the area of the parallelogram involved, see eq. (3.21). Taking all the cycles to be of similar size, the corresponding contact interaction mediating rare tau decays has a coefficient

\[ c_{\text{inst}} \sim \frac{8M_s^2 G_F}{\sqrt{2}} \frac{1}{M_s} \left( \frac{8\pi^2 l_s}{L_c} \right)^2 \left( e^{-A/(2\pi l_s^2)} \right). \]  

(4.6)
Figure 4: Instanton contribution to flavour violating processes such as $\tau \rightarrow e\mu\nu$. The string contribution is proportional to minus the exponential of the shaded area.

![Diagram](image)

Figure 5: Lower bound on the string scale as a function of the compactification length (in terms of the string length) coming from the contribution of gluon KK modes (solid line) and from instanton contributions (dashed line).

Using the latest Belle results on rare tau decays [26]

$$\text{Br}(\tau \rightarrow e\mu\nu) \leq 3.4 \times 10^{-7},$$

and considering small mixing angles so that the flavour eigenstates are close to the current eigenstates we find the bounds shown in figure 5.

As figure 5 shows, in models with intersecting D-branes, there are stringent bounds on the string scale from FCNC processes mediated by gauge boson KK modes when $l_s \ll L_c$, and by stretched string states when $l_s \gtrsim L_c$. These bounds seriously compromise non-supersymmetric models which, in order to avoid a hierarchy, need a low string scale. The bounds also imply that other phenomenological signals of these models, such as the presence of new U(1) gauge bosons arising from the gauge reduction $U(N) \sim SU(N) \times U(1)$ (see ref. [14]) are out of reach of present or near future experiments. (Note however that in some cases these extra U(1) gauge bosons can also have family non-universal gauge couplings [10], inducing FCNCs that could become relevant. See for example ref. [27].)
To summarise, we have discussed how FCNCs arise in models with intersecting D-branes, as a result of the family non universal couplings of the KK modes of the gauge bosons living in the world volume of the branes. We have also presented the equivalent full string calculation, finding good agreement with the field theoretic result. The string calculation gives a natural regularisation of the otherwise divergent KK sums. We also highlighted another source of flavour violation in these models which becomes important in the region of the parameter space (when the compactification scale is comparable to the string scale) where the KK contribution is subleading. A more detailed study of flavour physics in models with intersecting D-branes is currently in progress and will be presented elsewhere, but these initial results seem to indicate that non-supersymmetric models are strongly disfavoured for phenomenological reasons.

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