Scale dependence of quark mass matrices in models with flavor symmetries

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Numerical correlations between fermion masses and mixings could indicate the presence of a flavor symmetry at high energies. In general, the search for these correlations using low energy data requires an estimate of leading-log radiative corrections. We present a complete analysis of the evolution between the electroweak and the grand unification scales of quark mass parameters in minimal supersymmetric models. We take $M_t=180$ GeV and consider all possible values of $\tan\beta$. We also analyze the possibility that the top and/or the bottom Yukawa couplings result from an intermediate quasifixed point (QFP) of the equations. We show that the quark mixings of the third family do not have a QFP behavior (in contrast with the masses, the renormalization of all the mixings is linear), and we evaluate the low energy value of $V_{ub}$ which corresponds to $V_{ub}(M_X)=0$. Then we focus on the renormalization-group corrections to (i) typical relations obtained in models with flavor symmetries at the unification scale and (ii) a superstring-motivated pattern of quark mass matrices. We show that in most of the models the numerical prediction for $V_{ub}$ can be corrected in both directions (by varying $\tan\beta$) due to top or bottom radiative corrections. [S0556-2821(96)00923-X]

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I. INTRODUCTION

The recent observation of the top quark, with a mass around 180 GeV [1], allows a more complete analysis of the Yukawa sector of the standard model. In particular, it allows an evaluation of the running to higher energy scales of the parameters in that sector. The perturbative unification of the gauge couplings obtained in minimal supersymmetric (SUSY) scenarios suggests that the (nonsinglet) matter and gauge contents do not change up to energies around $10^{16}$ GeV. If that is the case, the first step to understand the flavor structure of the standard model is to evolve it up to those energies. We present in the first part of this article an updated and complete analysis of the evolution from low energies to the unification scale $M_X$ of the ten physical parameters in the quark mass matrices [six masses, three mixing angles, and the complex Cabibbo-Kobayashi-Maskawa (CKM) phase] in the minimal SUSY extension of the standard model (MSSM) [2]. We will study in detail the behavior of the mixing angles when the top and/or bottom Yukawa couplings $h_{t,b}$ are large at $M_X$. In this regime [3] the renormalization-group corrections focus any initial value of the coupling at $M_X$ to a narrow interval $\delta h_i$ [around $h_i(M_Z)=1.2$] at low energy: $(\delta h_i/h_i)(M_X)\ll(\delta h_i/h_i)(M_Z)$. We will analyze how this nonlinear evolution of the couplings [quasifixed point (QFP) behavior] affects the mixings with the third generation; in particular, we will find the low energy value of the mixing $s_{13}$ (with $V_{ub}=s_{13}e^{-i\delta_{13}}$) that corresponds to $s_{13}=0$ at $M_X$.

On the other hand, the observed pattern of fermion masses and mixings does not look accidental, and suggests a symmetry in the Yukawa matrices as the origin of the hierarchies. Obviously, such a flavor symmetry should be formulated at the unification scale. Although the Yukawa matrices contain more free parameters than physical observables, it is not easy to find simple structures that are able to accommodate without need of fine tuning the measured values of masses and mixings. As a matter of fact, the authors of Ref. [4] classify the symmetric quark mass matrices with a maximal number of texture zeros, and find that only five textures are acceptable experimentally. These matrices predict correlations between mass parameters that can be expressed in a simple (approximate) form; we will analyze how the correlations run from $M_X$ to low energies. We will also analyze a particular scenario [5] derived from the heterotic string which has been recently proposed as the only realistic possibility among the models within its class. In this scenario (and also in the model proposed in [6], with one more texture zero than those in the cases in [4]), the renormalization-group corrections could be essential to obtain a predicted mixing $V_{ub}$ within the experimental limits.

II. EVOLUTION OF QUARK MASSES AND MIXINGS

The quark Yukawa sector of the MSSM can be expressed in terms of the superpotential

$$P = h_{ij}^u H Q_i u_j^c + h_{ij}^d H' Q_i d_j$$

with $H=(H^+ H^0)$, $H'=(H'^+ H'^-)$, and $Q_i = (u_i d_i)$. In that sector there are ten independent physical parameters: once the Yukawa matrices are diagonalized, only the eigenvalues (three in the up and three in the down quark matrices) and the CKM matrix (with three mixing angles and a complex phase) appear in the Lagrangian. The procedure to obtain these parameters at $M_X$ from the pole masses and the low energy mixings is the following. For the heavier quarks, the perturbative pole mass $M_i$ is related to the running mass $m_i(M_t)$ in the modified minimal subtraction ($\overline{\text{MS}}$) scheme by a simple expression [7] (this change of scheme is numerically important due to the large size of $\alpha_s$). Taking $M_t=180$ GeV and $M_b=4.7$ GeV we obtain

$$m_b(M_t)=0.880M_b, \quad m_t(M_t)=0.946M_t.$$  

The running masses of the lighter quarks are given at 1 GeV [8]: $m_u=0.0056$ GeV; $m_d=0.0099$ GeV; $m_s=0.199$
\[ V_{ij} = \frac{h_{ij}^u}{\sin \beta}, \quad h_{ij}^d = \frac{h_{ij}^d}{\cos \beta}, \]  

where \( \tan \beta \) is the ratio of vacuum expectation values (VEV’s) giving mass to down and up quarks. At \( M_X \) we diagonalize the Yukawa matrices (we express the eigenvalues as \( h_i \), with \( i = u, c, \ldots \)) and find the CKM matrix. We will neglect corrections to mixing angles and light quark masses [11] proportional to the soft SUSY-breaking parameters, although these corrections can be significant in the large \( \tan \beta \) regime [12].

We present in Figs. 1 and 2 the evolution of masses and mixings for values of \( \tan \beta \) between 1.29 and 67.7, which correspond, respectively, to \( h_i/4 \pi \) and \( h_b/4 \pi \) equal to 0.25 at \( M_X \) (we analyze below in detail the quasifixed point regions). In Fig. 1 we plot \( r_i = m_i(M_X)/m_i(M_Z) \) for the six quarks (note that the ratios of masses and mixings at \( M_Z \) are also convenient to run the top quark mass down to that scale; we obtain \( m_t(M_Z) = 1.05 m_t(M_Z) \).

At \( M_Z \) we find the Yukawa couplings to the lightest neutral Higgs boson \( h \), which correspond to the Yukawas in the standard model (we assume \( m_h = M_Z \leq m_{\text{SUSY}} = 250 \text{ GeV} \))

\[ h_{ij}^u = \frac{h_{ij}^u}{\sin \beta}, \quad h_{ij}^d = \frac{h_{ij}^d}{\cos \beta}, \]  

\( V_{ij} \) being the CKM matrix at \( M_Z \). From \( M_Z \) to \( M_t \), these Yukawas evolve due to the gauge and the Yukawa interactions of the light quark and leptons (all of them included in our equations); between \( M_t \) and \( m_{\text{SUSY}} \) the top Yukawa corrections are also important. From \( m_{\text{SUSY}} \) up to \( M_X \approx 10^{16.2} \text{ GeV} \) (the unification scale that corresponds to \( \sin^2 \theta_W = 0.23195 \)) we include the interactions of the SUSY particles and the extra Higgs scalars (the standard model and MSSM renormalization group equations can be found in Refs. [9,10], respectively). The Yukawa couplings to the two Higgs doublets are (at \( m_{\text{SUSY}} \))

\[ r_{ij} = m_i(M_X)/m_i(M_Z), \quad r_{ij} = \arg V_{ij}(M_X)/\arg V_{ij}(M_Z), \]  

\( r_{ij} = \arg V_{ij}(M_X)/\arg V_{ij}(M_Z) \) (in the Maiani parametrization). In order to compare the relative renormalization of the mixings and different ratios of masses, we also plot \( R_u^3, R_d \) in Fig. 2, where \( R_u = [m_{u,c}(M_X)/m_i(M_X)]/[m_{u,c}(M_Z)/m_i(M_Z)] \) and \( R_d = [m_{d,c}(M_X)/m_i(M_X)]/[m_{d,c}(M_Z)/m_i(M_Z)] \). We take all the masses and mixings at \( M_Z \) in their central value \( (V_{us} = 0.221 \pm 0.003, \quad V_{cb} = 0.040 \pm 0.008, \quad V_{ub} = 0.0035 \pm 0.0015), \quad \) with \( M_t = 180 \text{ GeV} \) and \( \arg(V_{ub}) = -\pi/2 \).

We observe the following behavior.

(i) The evolutions of \( m_u \) and \( m_c \) coincide at the 0.04%; the same happens for \( m_d \) and \( m_l \) (0.06%) and for \( |V_{ub}| \) and \( V_{cb} (0.05\% \) )

(ii) The running of the Cabibbo mixing \( V_{us} \) and of the CKM phase (in \( V_{ub} \)) are smaller than the 0.03% and 0.07%, respectively. The evolution of masses and mixings is insensitive to the value of the complex phase.

(iii) The approximation [4]

\[ V_{ij}(M_X)/V_{ij}(M_Z) = \chi \quad (ij = us, cb, ub), \]  

\[ m_{u,c}(M_X)/m_{i}(M_X) = \chi^3, \]  

\[ m_{d,c}(M_X)/m_{i}(M_X) = \chi, \]  

is excellent (error smaller than 1%) for \( \tan \beta \approx 5 \). These expressions, with \( \chi = (M_X/M_Z)^{h/4 \pi} \), are obtained assuming that the top Yukawa coupling is dominant and constant between \( M_Z \) and \( M_X \).

As showed in [3], the low energy value of a large Yukawa coupling could be related to an intermediate QFP (previous to the Pendleton-Ross infrared fixed point [13]) of the renormalization-group equations. The effect of the QFP was thought to be a small correction, comparable to the corrections from the intermediate scale. However, what we found is that the QFP can be a significant source of the evolution of the Yukawa couplings and mixings, and that the low energy value of the coupling is not a constant, but evolves as \( \tan \beta \) changes.
would be to focus any large initial value (at $M_X$) of the coupling to a narrow region at $M_Z$. In the MSSM, the top coupling approaches a QFP value for $\tan\beta \sim 1$, and the bottom coupling may approach a QFP value for large $\tan\beta$. For a top mass on its upper experimental limit, both couplings would be attracted to QFP’s [14] of the equations. The first case [large $h_t(M_X)$ and low or moderate $\tan\beta$] has been analyzed in [15], where an approximate expression is found for the QFP value of the top quark mass:

$$m_t(M_t) = 196 \text{ GeV} \left\{1 + 2 [\alpha_t(M_Z) - 0.12] \sin\beta \right\} (6)$$

For $\alpha_t(M_Z) = 0.1165$ this expression gives $h_t(M_Z) = 1.18$.

In Tables I–III we show in some detail the behavior of quark masses and mixings near the QFP. Table I expresses the initial values (at $M_Z$) of masses and mixings, whereas in Table II we write the values at $M_X$ for different values of $\tan\beta$ which correspond to $h_t(M_X) = 2, 8$ [i.e., $h_t(M_X) / 4\pi = (0.16, 0.64)$ and $h_t(M_X) = 2, 8$ for $M_t = 180$ GeV. In Table III both couplings are equal to 2 and/or 8 at $M_X$ (this implies $M_t = 190–220$ GeV and $\tan\beta = 60–69$). In all cases, the large top (and/or bottom) Yukawa coupling at $M_X$ is attracted to a QFP value around 1.2: ($\delta h_t / h_t)(M_X) = 0.06$ for any value of $h_t / 4\pi(M_X)$ larger than 0.32. Although for such large values of the couplings this attraction is quite effective, it is not complete; $h_t(M_Z)$ has some dependence on the initial (large) value of $h_t(M_X)$, giving variations that can increase its actual value by 10% with respect to the (approximate) QFP value given above. A more effective attraction would be obtained for smaller $h_t(M_X)$ or larger $M_X$. The evolution of the mixing angles for Yukawa couplings near the QFP value presents the following features.

(i) The running of the mixings with the third family (and also the Cabibbo mixing) is highly linear, in the sense that $(\delta V_{ij} / V_{ij})(M_X) = (\delta V_{ij} / V_{ij})(M_Z)$, with $ij = us, ub, cb$. This fact means that the angles do not have a QFP behavior. For example, for $h_t(M_X) = 8$ the interval $V_{ub}(M_Z) = 0.002–0.005$ evolves to $V_{ub}(M_X) = 0.00135–0.00337$.

The evolution of $V_{us}$ is still smaller than 0.04%. The difference between the running of $V_{cb}$ and $V_{ub}$ and of the up and charm (down and strange) Yukawas, smaller than 0.02% and 0.04% (0.3%), respectively, do not grow when increasing the Yukawa couplings at $M_X$.

(ii) The mixings with the third family $V_{ib}$ and the ratios $\sqrt{m_{u,e}} / m_t$ and $\sqrt{m_d} / m_t$ tend to zero at $M_X$ when $h_t(M_X)$ and/or $h_b(M_X)$ increase. However, when increasing for example $h_t$, the three quantities decrease at different rates. As a consequence, it will be always possible to correct a relation between masses and mixings by varying $\tan\beta$ and going to the adequate (top or bottom) QFP region (see next section).

(iii) To illustrate the size of the nonlinear effects (first point above), we will consider the case when $V_{ub}$ vanishes at $M_X$. The running from $M_X$ to $M_Z$ generates then a nonzero mixing. In Fig. 3 we plot $V_{ub}(M_Z)$ for $\tan\beta$ between 1.29 and 67.7, with the fermion masses and the rest of mixings in their central values. We find a value much smaller than the experimentally preferred, although it grows when $\tan\beta$ decreases [i.e., for a fixed $m_t$, $h_t(M_X)$ increases]. We also show in Table II the low energy value of $V_{ub}$ which corresponds in each case to a vanishing mixing at $M_X$. Note that

| $V_{ub}(M_Z)$ | 0.218–0.224 |
| $\sqrt{m_u} / m_t(M_Z)$ | 0.0609 |
| $\sqrt{m_t} / m_t(M_Z)$ | 0.224 |
| $V_{ab}(M_Z)$ | 0.00135–0.00337 |
| $\sqrt{m_u} / m_t(M_Z)$ | 0.00223 |
| $\sqrt{m_t} / m_t(M_Z)$ | 0.0355 |
| $V_{cb}(M_Z)$ | 0.0216–0.0324 |
| $\sqrt{m_u} / m_t(M_Z)$ | 0.0366 |
| $\sqrt{m_t} / m_t(M_Z)$ | 0.159 |
| $V_{ab}(M_Z)$ | $6.56 \times 10^{-6}$ |

**TABLE II.** Value of $V_{ub}(M_Z)$ for $\tan\beta$ which correspond to large $h_t$ or $h_b$ at $M_X$ ($M_t = 180$ GeV). In the last line we write the value of $V_{ub}(M_Z)$ which would correspond to $V_{ab}(M_Z) = 0$. We include the value of $h_t$. 

| $\tan\beta$ | 1.22 |
| $h_t(M_Z)$ | 1.33 |
| $h_b(M_Z)$ | 0.028 |
| $h_t(M_Z)$ | 0.016 |
| $h_b(M_Z)$ | 0.024 |
| $h_b(M_X)$ | 0.013 |
| $V_{us}$ | 0.218–0.224 |
| $\sqrt{m_u} / m_t(M_Z)$ | 0.0609 |
| $\sqrt{m_t} / m_t(M_Z)$ | 0.224 |
| $V_{ub}$ | 0.00135–0.00337 |
| $\sqrt{m_u} / m_t(M_Z)$ | 0.00223 |
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| $\sqrt{m_t} / m_t(M_Z)$ | 0.159 |
| $V_{ab}(M_Z)$ | $6.56 \times 10^{-6}$ |
V_{ub}, the smallest mixing in the CKM matrix (s_{13} in the Maiani parametrization), is a physical parameter whose zero value at one loop is not protected by any symmetry. A flavor structure giving $V_{ub} = 0$ at $M_X$ would be characterized by $V_{ub} (M_Z) = 10^{-5}$.

Although we do not intend to discuss here the lepton sector (the possible bottom–tau unification has been analyzed in great detail in [16]), we include the tau Yukawa coupling at different scales in Tables II and III (as mentioned in the Introduction, a complete analysis of the large tan$\beta$ regime requires an evaluation of soft SUSY-breaking corrections [12]).

### III. EVOLUTION OF FLAVOR RELATIONS

We proceed now to study how the evolution from $M_X$ to $M_Z$ affects the relations between quark mass parameters obtained in models with flavor symmetries at the unification scale. First we will consider the five symmetric patterns with a maximal number of texture zeros found in Ref. [4]. These patterns depend on seven complex parameters which, after phase redefinitions, are reduced to seven moduli and two phases (three phases in solution 2 in [4]). The approximate analysis shows that it is possible to adjust the experimental masses and mixings without need of fine tuning. In particular, the absence of significant cancellations implies that the only role of the 2 complex phases in each pattern is to generate the CKM phase. As a consequence, the seven moduli fit the six quark masses and three mixings giving two relations. One relation [17] is shared by all the cases (solutions 1–5 in [4]):

$$V_{us} = \sqrt{\frac{m_d}{m_s}}$$  \hspace{1cm} (7)

(with complex corrections of modulus $\sqrt{m_u/m_c}$ in solutions 1, 2, 4, 5). The second relation is

$$\frac{V_{ub}}{V_{cb}} = \sqrt{\frac{m_u}{m_c}}$$  \hspace{1cm} (8)

for solutions 1, 2, 4 and

$$V_{ub} = \sqrt{\frac{m_u}{m_t}}$$  \hspace{1cm} (9)

for solutions 3 and 5 [in the last case there are complex corrections of order $(m_t/2m_t)V_{cb}^2 \approx 20\%$]. The masses and the rest of the mixings can be adjusted to their central values, with an arbitrary CKM phase.

Relations (7)–(9) are established at $M_X$, and one has to evolve the experimental quantities up to that scale in order to decide if they are acceptable. The running of the first two relations, however, is just a 0.2\% (smaller than corrections to the approximate diagonalization of the matrices). Taking the

![FIG. 3. Value of $V_{ub}(M_Z)$ that would correspond to a zero value of $V_{ub}(M_X)$, plotted for different values of tan$\beta$. The masses and the mixings $V_{us}, V_{cb}$ are taken at their central values.](image-url)
masses in their central values we have $\sqrt{m_{\tau}/m_\tau}=0.223$ and $\sqrt{m_{\mu}/m_\mu}=0.064$, which compare well with the data ($V_{13}=0.221\pm0.003$ and $V_{13}/V_{cb}=0.08\pm0.02$ [8]). The relation $V_{ub}=\sqrt{m_{\tau}/m_\tau}$ suffers sizable renormalization-group corrections. At $M_Z$ we have $V_{ub}=0.0035\pm0.0015$ and $\sqrt{m_{\mu}/m_\mu}=0.76\sqrt{m_\mu(1\, \text{GeV})}/M_\tau=0.0040$. The running from $M_Z$ to $M_X$ can be expressed in terms of the ratio

$$r=\frac{\sqrt{m_{\tau}/m_\tau(M_X)}}{\sqrt{m_{\mu}/m_\mu(M_Z)}}/\frac{V_{ub}(M_X)}{V_{ub}(M_Z)}$$

that we plot in Fig. 4 for different values of $\tan\beta$. Note that for $\tan\beta<0.25, V_{13}$ diminishes less than $\sqrt{m_{\mu}/m_\mu}$ (i.e., $r<1$), while for larger values of $\tan\beta$ we observe the opposite behavior. If the masses and mixings were known with more accuracy, this fact could be used to correct the prediction

$$V_{ub}=0.0040r$$

in the preferred direction.

The different running of the mixings and the ratios of quark masses involving the third family would also affect the symmetric texture proposed in [6] (with one more zero than the textures in [4]). Those matrices predict $V_{cb}=\sqrt{m_\tau/m_\tau}$, a value that seems too large: the values of the masses at $M_Z$ suggest $V_{cb}=0.76\sqrt{m_\mu(1\, \text{GeV})}/M_\tau=0.066$, while the experimental upper bound is 0.048. Since the evolutions of $V_{ub}$ ($m_{\tau}$) and $V_{cb}$ ($m_\tau$) coincide, the running from $M_X$ will be simply expressed by the same factor $r$ in Fig. 4:

$$V_{ub}=0.066r$$

and the relation would be experimentally acceptable for $\tan\beta\leq1.4$ (with $M_t=180\, \text{GeV}$).

We will finally analyze a pattern of quark matrices derived from the heterotic string. The matrices have been proposed [5] as the only realistic possibility in a class of models compactified in the Tian-Yau manifold. Their structure is

$$M_u=\begin{pmatrix} 0 & D_0 & C_0 \\ D_0 & 0 & B_0 \\ C_0 & B_0 & A_0 \end{pmatrix}, \quad M_d=\begin{pmatrix} D_0'' & 0 & 0 \\ 0 & C_0' & B_0'' \\ 0 & B_0' & A_0' \end{pmatrix}. \tag{13}$$

By a redefinition of the quark fields we can put these matrices in a more convenient form:

$$M_u=\begin{pmatrix} 0 & \tilde{D} & C \\ \tilde{D} & 0 & B \\ C & B & A \end{pmatrix}, \quad M_d=\begin{pmatrix} D' & 0 & 0 \\ 0 & C' & 0 \\ 0 & B' & A' \end{pmatrix}. \tag{14}$$

where $\tilde{D}$ and $\tilde{B}$ are complex and the rest of the parameters are real and positive. As we will see, these eight moduli and two phases can fit all the masses and the larger mixings to their central values and predict acceptable (but large) values for $V_{ub}$ and a nonzero (but small) complex CKM phase. The approximate diagonalization gives

$$m_t=A, \quad m_c=B^2/A, \quad m_u=\frac{2ABC\tilde{D} - A^2\tilde{D}^2}{AB^2}, \tag{15}$$

$$m_b=A', \quad m_s=C', \quad m_d=D', \tag{16}$$

and a CKM matrix (in the Maiani parametrization) with

$$V_{us}=\begin{pmatrix} BC - AD \\ B^2 \end{pmatrix}, \tag{17a}$$

$$V_{cb}=\begin{pmatrix} \tilde{B} & B \\ A' & A \end{pmatrix}, \tag{17b}$$

$$|V_{ub}|=\left|\frac{(BC-\tilde{A}D)\tilde{B}}{AB^2} - \tilde{D}\right|/B. \tag{17c}$$

In terms of physical quantities we have

$$V_{ub}=\left( V_{us}V_{cb} + V_{12}\sqrt{m_\tau/m_\tau} - m_uV_{cb} \right) + \frac{m_uV_{cb}e^{i\alpha}}{2m_cV_{12}e^{i\beta}}, \tag{18}$$

where $\alpha$ and $\beta$ are independent complex phases (the dominant phase $\alpha$ is related to the phase of $\tilde{B}'$). At $M_Z$, for masses and mixings in their central values, the relation reads $V_{13}=0.0088+0.0146e^{i\alpha}-0.0004e^{i\beta}$. Then it seems that the small value of $V_{ub}$ requires a cancellation between the first two terms, with a best value $|V_{ub}|>0.0054$ (for $\alpha=\pi$). Renormalization-group corrections affect this relation due to the different running of $V_{cb}$ and $\sqrt{m_\mu/m_\mu}$, with the total effect captured again by the factor $r$ plotted in Fig. 4:

$$V_{ub}=0.0088 + r 0.0146e^{i\alpha} - 0.00034e^{i\beta}. \tag{19}$$

For low values of $r$, the lower bound for the predicted value of $|V_{ub}|$ decreases. For example, for $\tan\beta=1.5$ we have

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1We suppress an antisymmetric entry proportional to the VEV’s of an extra Higgs doublet present in the model [5] since its presence would require a detailed analysis of flavor-changing neutral currents.
$|V_{ub}| > 0.002$ and a CKM phase $\pi/2 \leq \delta_{13} \leq 3\pi/2$, whereas $|V_{ub}| < 0.005$ would imply $\tan\beta \approx 30$ (for all the quark masses and the rest of mixings in their central values). Note that for smaller values of $V_{cb}$, these bounds are relaxed.

**IV. CONCLUSIONS**

The observed value of $M_t$ implies that $h_i$ is the dominant term in the renormalization-group equations at large scales. As a consequence, the corrections to the quark masses lose universality and there appear nontrivial corrections to the CKM mixings of the light quarks with the third family. In addition, the low energy value of $h_i$ could be related to a QFP of the equations: any large value of $h_i(M_X)$ seems to converge to a narrow interval around 1.2 at QFP of the equations: any large value of $h_i(M_X)$ seems to converge to a narrow interval around 1.2 at QFP. The MSSM with $M_t = 180$ GeV this forces a low value of $\tan\beta$, whereas an analogous situation occurs for $h_i$ in the large $\tan\beta$ regime. For $M_t$ around 200 GeV and large $\tan\beta$, both low energy Yukawa couplings would result from any large value of the couplings at $M_X$ (a large value of $\tan\beta$ could be also motivated by the possibility to relax the $K_b$ anomaly [18]). In this framework, we perform an updated (with the new data for $M_t$) and complete (all values of $\tan\beta$) analysis of the evolution from $M_Z$ and $M_X$ of all the physical observables in the quark Yukawa sector of the MSSM. We study in detail the behavior of the smallest CKM mixing $V_{ab}$ in the top and/or bottom QFP regions, and we show that the evolution is linear: $\delta V_{ab}/V_{ab}(M_Z) = \delta V_{ab}/V_{ab}(M_X)$.

To illustrate the size of the nonlinear corrections we analyze the value of $V_{ab}(M_Z)$ which corresponds to $V_{ab}(M_X)$; we obtain $V_{ab}(M_Z) \approx 10^{-5}$ (this value increases going to nonperturbative values of $h_i$ at $M_X$, i.e., lowering $\tan\beta$).

Then we analyze the renormalization-group corrections to fermion mass relations which appear in models with flavor symmetries at $M_X$. In particular, we discuss the relations obtained for symmetric mass matrices with a maximal number of zeros and in a superstring-motivated model. We show that in some relations the corrections can be numerically important (they are essential in some of the cases) and that they depend quite strongly on $\tan\beta$. In particular, for the relations analyzed the corrections can be expressed in terms of the ratio $r$ in Eq. (10). We find it remarkable that, for a fixed $M_t$, $r$ goes to zero decreasing $\tan\beta$ and grows $(r > 1)$ for $\tan\beta$ large ($\tan\beta \approx 62$). If the masses and mixings were measured with more accuracy, this fact could be used to conveniently correct the relations by varying $\tan\beta$, whereas if the Higgs sector of the MSSM were observed and $\tan\beta$ fixed, it could be used to exclude some of the quark mass matrix models.

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