Time Domain Analysis of Dielectric-Coated Wire Antennas and Scatterers

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Abstract—DOTIG3 is a computer code developed to calculate, in the time domain, the interaction of transient electromagnetic pulses (EMP) with perfect electric conductor structures modeled by thin wires. The numerical procedure is based on the solution of the electric field integral equation (EFIE) by the moment method. This paper describes an extension of the DOTIG3 code that enables it to calculate radiation and scattering of transient electromagnetic pulses by homogeneous, dielectric coated thin wires. The modifications carried out to extend the application of DOTIG3 to coated wires are described and the results obtained compared, by way of Fourier transformation, with those obtained by other authors working in the frequency domain, and with experimental data. There is an excellent agreement between the predictions of the DOTIG3 code and other authors’ results.

I. INTRODUCTION

DIELECTRIC coatings are often used for packaging, or to isolate a wire antenna. They can also be used to modify the scattering or radiating properties [1], [2], or to find the equivalent cylindrical antennas of one with a noncircular cross section [3]. The thickness of the coatings is usually similar in size to the wire radius [4]. This paper describes an extension of DOTIG3 [5]–[7] to study, in the time domain, the transient analysis of dielectric-coated thin wires. Dielectric-coated wires have been treated, in the frequency domain, by a number of authors [1], [2], [4], [8]–[10]. The method used in DOTIG3 is based on the quasi-stationary approximation of the electric field in the coating planes transverse to the wire axis, proposed in [1] for frequency domain analysis.

The paper is organized as follows. In Section II we briefly describe the basics of the computer code DOTIG3 for the analysis, in the time domain, of the interaction of arbitrary transient electromagnetic waves with perfect electric conductor structures (PEC) modeled by a set of interconnected thin wires. Section III continues with the theoretical background and the required modifications to extend the application of DOTIG3 to study the radiation and scattering of transient electromagnetic pulses by coated thin wires. Some results are presented in Section IV. The current distribution on a coated monopole, and the RCS from a coated wire, is calculated by DOTIG3, and compared with other authors’ experimental and/or numerical data.

Fig. 1. Geometry for the thin-wire electric field integral equation.

II. BRIEF DESCRIPTION OF DOTIG3

DOTIG3 solves, using a marching-on-in-time procedure of the method of moments, the thin-wire time-domain electric field integral equation (TD-EFIE) [6], [7]

\[
\ddot{z} \cdot \vec{E}(z,t) = \frac{1}{4\pi\varepsilon_0} \int_0^L \left( \frac{\ddot{z} \cdot \vec{z}'}{c^2 R} + \frac{\ddot{z} \cdot \vec{R}}{c R^2} \frac{\partial}{\partial z'} I(z',t') \right) dz' + \frac{\ddot{z} \cdot \vec{R}}{c R^2} \frac{\partial}{\partial z'} q(z',t') dz'
\]

where \(L\) is the length of the wire, \(\ddot{z}\) is a vector tangent to the wire axis at position \(z(t'') = z', I(z',t')\) is the unknown current at source point \(z'\) at retarded time \(t' = t - R/c\) defined as

\[
I(z',t) = a \int_0^{2\pi} J_z(z',\phi,t) d\phi
\]

where \(J_z\) is the \(z\) component of electric surface-current density induced on the wire, \(a\) the radius of the wire, \(c\) the light velocity in the free space, \(\varepsilon_0\) the permittivity of the free space, and \(q(z',t')\) is the charge distribution that can be expressed in terms of the current using the equation of continuity. Observe that the source points \(z'\) are located on the surface of the wire and the field points \(z\) on the axis (see Fig. 1). We refer to (1) as a thin-wire extended-boundary-condition EFIE [7], [11]–[13].

To solve (1) DOTIG3 uses the point-matching form of the method of moments. In this procedure, (1) is discretized in space and time, and the unknown distribution of the induced current at a time step is expressed in terms of previously
calculated current values, and in terms of the known incident field on the axis of the wire. The delta functions used as weighting functions are situated at the ends of the space and time intervals to avoid discontinuities in the magnitude of the current. The spatial and temporal variation of the current is represented by a two-dimensional Lagrangian interpolation of order two in each dimension (space and time) as the basis function. This allows us to transform the integral (1) into the linear system of equations (in matrix notation) [14]–[17]:

\[ E_j^\theta + E_j^z = \hat{Z} I_j. \]  
(3)

Using (3), current \( I_j \) at time \( t_j \) is calculated from the elements \( E_j^\theta \) of the tangential electric field scattered by currents of previous times and the elements \( E_j^z \) of the tangential applied electric field at the observation point and at time step \( t_j \). Observe that because the axial component of the total electric \( E^t = E_j^\theta + E_j^z \) is forced to be null on the axis of the wire, it must be evaluated at the times \( t_j = j \delta t + a/c \) (\( j = 1 \) to \( N_t \)), \( \delta \) being the duration of the temporal intervals [5]. The matrix \( \hat{Z} \) is a matrix of interaction, whose elements are time independent. They depend only on the geometry and on the electromagnetic characteristics of the structure.

### III. THEORY FOR DIELECTRIC-COATED WIRES

Consider a perfectly conducting wire of circular cross section and radius \( a \), coated with a thin cylindrical homogeneous dielectric coating of thickness \( b - a \) and permittivity \( \varepsilon \). As long as \( b - a \) and \( \varepsilon \) are not too large, any electric field in the coating can be considered radial throughout the coating [1], [2]. This radial electric field produces a radial polarization current within the coating and two layers of polarization charges on the two cylindrical surfaces of the coating of equal magnitudes and opposite signs. Applying Gauss’ theorem [18], the polarization charge per unit length on the inner surface (radius \( a \)) of the coating is

\[ q_a(z', t) = -\epsilon_0 \varepsilon \varepsilon_0 * q(z', t) \]  
(4)

and on the other (radius \( b \)) is

\[ q_b(z', t) = (\epsilon - \epsilon_0)/\epsilon_0 * q(z', t - b/a) \]  
\[ = -q_a(z', t - b/a) \]  
(5)

where \( q \) is the free charge per unit length on the wire (assumed uniformly distributed around the thin wire). Observe that in (5), the time delay \((b - a)/v\) needed for calculating the polarization charge at radius \( b \) due to the free charge at radius \( a \) was taken into account, where \( v \) is the light velocity in the dielectric medium.

The total charge \( Q \) per unit length will be (adding free and polarization charges) at \( r = a \)

\[ Q_a = q(z', t)/\varepsilon_\varepsilon \]  
(6)

and at \( r = b \)

\[ Q_b(z', t) = (\varepsilon_\varepsilon - 1)/\varepsilon_\varepsilon * q(z', t - (b - a)/v) \]  
(7)

where \( \varepsilon_\varepsilon \) is the relative permittivity.

The influence of the coating on the electromagnetic field can now be reduced to the polarization charges (6) and (7) situated in the vacuum and with their corresponding time delays.

To include these contributions in (1) we must take into account that this equation has been derived from the two-potential expression for the electric field \( E^t \) scattered by the wire:

\[ \epsilon_0 E^t = -\frac{1}{c^2} \frac{\partial A}{\partial t} - \nabla \phi \]  
(8)

where the magnetic vector potential and electric scalar potential are defined as

\[ A = \frac{1}{4\pi} \int_S \frac{\mathbf{J}(\mathbf{r}, t')}{R} dS', \]  
(9)

\[ \phi = \frac{1}{4\pi} \int_S \frac{\sigma(\mathbf{r}, t')}{R} dS'. \]  
(10)

In (1), the first term in the integral represents the scattered field contributed by the current distribution on the wire. The other two terms represent the scattered field due to charge distribution. Since the field due to the radial polarization current can be discarded [1], [2], the only modification due to the electric coating is an addition to the second and third terms on the right of (1) contributed by the extra charges given by (4) at radius \( a \) and (5) at radius \( b \). Adding these contributions transforms (1) into

\[ \hat{z} \cdot E^t(z, t) = \frac{1}{4\pi\epsilon_0} \int_0^L \frac{\hat{z}}{c^2 R_a} \partial_t q(z', t - \frac{R_a}{c} dz') \]  
\[ + \frac{1}{4\pi\epsilon} \int_0^L \left( \hat{z} \cdot \frac{\hat{R}_a}{c R_a^2} \partial_t I(z', t - \frac{R_a}{c}) \right) dz' \]  
\[ - \frac{\hat{z} \cdot \hat{R}_a}{R_a^2} q(z', t - \frac{R_a}{c}) \int_0^L \eta I(z', t - \frac{R_a}{c} - \frac{(b - a)}{v}) dz' \]  
\[ + \frac{(\epsilon_\varepsilon - 1)}{4\pi\epsilon} \int_0^L \left( \hat{z} \cdot \frac{\hat{R}_b}{c R_b^2} \partial_t I(z', t - \frac{R_b}{c} - \frac{(b - a)}{v}) \right) dz' \]  
(11)

where

\[ R_a = \{(z - z')^2 + a^2\}^{1\over 2}, \]  
\[ R_b = \{(z - z')^2 + b^2\}^{1\over 2}. \]  
(12)

Because the electric field in the coating has been assumed to be radial throughout the coating, this treatment is, strictly speaking, only a very good approximation. This is because although the conductivity of the core is perfect, there is an absence of axial component strictly at its surface only; a small but increasing axial component will appear as one moves away from this surface within the coating. Popovic et al. have found that the method gives acceptable results for coatings up to about two core radii \((b \leq 3a)\) in thickness and dielectric constants \(\varepsilon_\varepsilon \leq 10\) [1].
IV. RESULTS

In this section we present results obtained using DOTIG3 for studying two different coated thin wires. The parameters of the structures were chosen in order to compare with results previously obtained by other authors, working in the frequency domain [1].

The first structure studied was a monopole antenna of length \( L = 0.125 \) m and radius \( a = 3.18 \) mm, covered with a thin dielectric coating of thickness \( b - a = 3.18 \) mm and relative permittivity \( \varepsilon_r = 3.2 \). The monopole was mounted on a perfectly conducting plane and fed at its base by a source voltage in \( V \) of the form

\[
V(t) = \exp(-g^2(t - t_{\text{max}})^2)
\]

(13)

with \( g = 2.5 \times 10^9 s^{-1} \) and \( t_{\text{max}} = 8.6 \times 10^{-10} s \). The structure was modeled by DOTIG3 as a linear dipole of length \( 2sL \) fed at its center by \( 2sV(t) \). The same geometry was also studied in the case where the wire was not covered by any dielectric medium (\( \varepsilon_r = 1 \)).

As the problem was treated directly in the time domain, we first present the source current versus time in Fig. 2, in the cases of \( \varepsilon_r = 3.2 \) (continuous line) and \( \varepsilon_r = 1 \) (discontinuous line). Observe that there is a shift to the right of the coated-wire data, that is, the pulse current traveling along the antenna is reflected later than in the case of the bare wire. The coated monopole behaves as if it were longer than the bare wire; the effect of the coating is to increase the effective length of the antenna.

In order to compare the DOTIG3 results with those obtained in the frequency domain by Popovic [1], the ratio of the Fourier transforms of the current along the wire and the source voltage was computed. The conductance of the dielectric coated antenna and of the bare antenna is plotted in Fig. 3 versus frequency. Fig. 4 shows the susceptance of the same antenna. NEC [19] results for the bare monopole, and Popovic’s results for the bare and coated wires are plotted for comparison. Although the agreement between DOTIG3 and Popovic’s results is good, the differences increase as \( \omega \) becomes larger. One possible explanation is that the marching-on-in-time method uses a fixed space discretization, while frequency-domain methods do not. In any case, as the spectral energy of the signal is concentrated in the lower frequencies, it is not necessary to calculate high-frequency constituents with great accuracy, if we are interested only in the time domain solution.

The current along the monopole, for a frequency \( f = 600 \) MHz, is plotted in Fig. 5 versus the length measured along the wire from its base. Continuous lines show the real and imaginary parts of the current calculated by DOTIG3, the crosses are Popovic’s numerical results, and the squares are experimental data measured by Lamensdorf [20]. It can be seen that there is a good agreement between DOTIG3 and the other authors’ results.

As a further example, the broadside RCS expressed in db, is shown in Fig. 6 for a straight thin wire coated with various dielectric media (with \( \varepsilon_r = 1.2 \) and 5.3). The length of the wire was 1 m, and its outer and inner radii were \( b = 1.4 \) cm,
This paper describes an extension of the DOTIG3 code for studying the interaction of transient electromagnetic signals with perfectly conducting wires covered with a thin homogeneous dielectric coating. The approximations proposed by other authors [1], [2] for treating the problem in the frequency domain have been translated to the time domain, taking into account, in the variables involved, the time delays due to the finite velocity of light propagation. The EFIE is formulated, for the coated wire, considering the effect of the polarization charges on the surfaces of the coating, and solved, directly in the time domain, by the moment method. The treatment is approximate, and the results are valid when the thickness of the coating is up to a wire radius of one or even two, and the relative permittivity $\varepsilon_r$ is less than about 10.

The new code has been tested against experimental [19] and numerical [1], [2] results, for the current distribution on a monopole antenna. There is an excellent agreement between DOTIG3 and other authors’ data. Results from DOTIG3 are also presented for broadside scattering by a single wire coated with different dielectric media. It was seen that the coating, although thin, influences the properties of antennas and scatterers. It has the effect of increasing the effective length of the wires.

V. CONCLUSIONS

DOTIG3 is a computer code for the study in the time domain, of the interaction of transient electromagnetic signals with perfectly conducting structures modeled by thin wires. The structures are inserted in a homogeneous dielectric medium. The numerical method used is based on the solution of the time-domain electric field integral equation (EFIE).

REFERENCES

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