measured at X-band with an HP 8719D vector network analyser. The measured raw S-parameters were processed using the new method and the traditional method. Both the isolator and the phase shifter have small reflections at their ports at X-band so that use of the proposed method is appropriate. Calibrated forward transmission coefficients of the test devices are shown in Figs. 1 and 2. It is found that the results from the two methods for the isolator agree closely, which means that both methods are adequate for this device. It is also seen that the data from both methods for the phase shifter are in general agreement, but some points with transmission greater than 0dB from the traditional method are unrepeatable while the data from the new method are always below 0dB. Therefore, more reliable and accurate transmission parameters of two-port devices with small reflections can be obtained with the new method at no cost, when compared with the traditional method.

Conclusions and discussions: For the first time, it has been shown theoretically and experimentally that accurately calibrated transmission parameters of a two-port device with small reflections can be obtained from only two $S$-matrix measurements: through-connected system without the device and loaded system with the device. For the case of no reflection at one of the device ports, the proposed equations become exact. This conclusion is valid for reciprocal and nonreciprocal devices. The presented method is useful for the characterization of both non-reflecting and reflecting devices because the new method can always guarantee higher accuracy than the traditional method, both using two $S$-matrix measurements.

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References

Simple derivation of scattering matrix for TLM nodes

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The problem of obtaining the scattering matrix for a transmission line modelling node is considered from a different point of view. For a specific incident pulse, the node lines are separated into common and uncommon lines. This separation allows the voltage at uncommon lines to be determined by simple circuit considerations. The method is illustrated with its application to a thin-wire node.

Introduction: One of the most outstanding features of the transmission line modelling (TLM) method consists of its versatility for treating a great variety of problems by including modifications in the symmetrical condensed node (SCN) proposed by Johns in [1]. As an example, the presence of wires has been considered by adding open and short-circuited stubs to take into account the capacitance and inductance introduced by the wire [2]. Unfortunately, designing a modified node to describe a special geometry is much easier than obtaining its corresponding scattering matrix. The imposition of Maxwell equations is perhaps the most direct and elegant way of establishing equations for deriving this matrix, but they are usually very difficult to solve. If capacitive and inductive stubs are included, the task is almost prohibitive except for a few cases. An alternative way to determine the scattering matrix of a node is presented in [3], in which a set of series and shunt nodes are proposed. Starting from a common voltage for a shunt node and a common current for a series node, the pulses reflected to the pair e lines common to both nodes can be obtained. The object of this Letter is to present a new and very simple method to obtain the scattering matrix for general TLM nodes. In a sense, the method can be considered as a combination of the two aforementioned methods as it has the advantages of both: the intuitiveness of considering Maxwell equations to obtain reflected pulses corresponding to a unique incident pulse, and the ease of using simple circuits of bidimensional nodes.

Scattering matrix from new point of view: An almost general TLM node useful for dealing with a great number of geometries can be constructed by considering the standard SCN with stubs proposed in [1] but with an arbitrary value for the characteristic impedance $Z_c = 1/V$ for each one of its 18 transmission lines. In this situation, we consider, for example, a unitary pulse incident on line 1. This pulse is related to the $E_c$ and $H_c$ propagating along the y-direction, that is to say, to the $\pi$-component of Ampere's law and the $\zeta$-component of Faraday's law. A shunt node defining a common $V$, voltage and a series node that defines a common $I$, current (Fig. 1) can be associated to Ampere's and Faraday's laws, respectively. From the shunt node, it is clear that the incident pulse through line 1 produces pulses reflected to lines 1, 2, 9, 12, and 13 while, from the series node, the incident pulse causes reflected pulses to appear at lines 1, 3, 11, 12, and 18.

![Fig. 1 $E_s$ shunt and $H_s$ series node](image)

To determine the amplitude of pulses reflected to lines 1 and 12 (which we will refer to as common lines since they appear in both nodes of Fig. 1), the two components of Maxwell equations mentioned above must hold. This means that both circuits are to be considered and so they are somehow interconnected and difficult to deal with. In contrast, to determine the amplitude of the pulse reflected to those lines that only appear in the shunt node, lines 2, 9, and 13, only the $\pi$-component of Ampere's law is to be considered (uncommon lines). This means that all the scattering information concerning these uncommon lines is contained solely in the $E_s$ shunt node and, therefore, the pulses scattered to them can be directly obtained by determining the transmission coefficient corresponding to an admittance $Y_s$ for the incident line and a total load admittance that is the shunt equivalent of the rest of lines in the node.

Similarly, only the $\zeta$-component of Faraday's law must be taken into account to obtain the amplitude of pulses reflected to lines 3, 11, and 18, and so their values can be obtained by considering.
only the geometry of the series node. A simple transmission coefficient determines the amplitude of the voltage transmitted to the series load, the voltage at lines 3, 11, and 18 being calculated by using a simple voltage divider.

At this point, only the pulses at common lines 1 and 12 remain unknown. However, they can be directly determined by imposing charge conservation at the shunt node and continuity of electrical potential at the series node. Repeating this procedure for all the lines in the node allows us to obtain the scattering matrix of the node. It is worth noting that, when the unitary incident pulse corresponds to a capacitive stub, only one component of Amperé's law is involved, i.e. only one shunt node is to be considered. In consequence, there are no common lines, because there is no series node, and all the reflected pulses can be obtained from the transmission and reflection coefficient of the shunt node. Of course, the same holds for a unitary pulse entering the node through an inductive or short-circuit stub.

![Fig. 2 Wire model](image)

**Example:** To illustrate the procedure, we consider a node for modeling a thin wire oriented along the x-direction. As discussed in [2], the modified node is formed of twelve link lines of characteristic impedance $Z_0$, for modelling the medium, and a series circuit of three extra lines (Fig. 2) that is connected to the shunt node for $E_r$.

For a unitary pulse traveling towards the centre of the node through line 1, the scattered pulses are obtained as follows. For the $E_s$ shunt node, the admittance of the incident line is $Y_s$, while the load admittance is $Y_L = 3Y_s + 1/(2Z_L + Z_0)$. The transmission coefficient $\tau$ is

$$\tau = \frac{2Y_s}{Y_1 + Y_L} = \frac{2Z_L}{2(Z_L + Z_0 + Z_s)/4}$$

and so pulses at the uncommon lines are

$$V_1^f = V_0^r = \tau, \quad V_2^0 = -V_1^r = \tau \frac{Z_L}{2Z_L + Z_s}$$

For the series node (Fig. 1 with $Z_0$), the impedance at the incident line is $Z_s$ and the load admittance is $3Z_0$ so the transmission coefficient is $1/2$ and the portion that goes to uncommon lines is $V_0^f \sim -V_1^r = 1/2$. Finally, charge conservation and continuity of electrical potential allows the voltage at common lines $V_2^f$ and $V_0^r$ to be determined.

**Conclusions:** A very simple method to obtain the scattering matrix for difficult TLM nodes has been presented. For a given unitary pulse, a pair consisting of a series and a shunt node can be defined. Two lines appear in both nodes (common lines) while the rest of the lines appear only in the series or the shunt node (uncommon lines). Voltage pulses transmitted to the uncommon lines are directly obtained from circuit considerations while voltage pulses at common lines are obtained by imposing charge conservation and continuity of electrical potential.

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**References**


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**Fuzzy inference systems and artificial neural networks for continuous piecewise multilinear interpolation**

L.C. Westphal

Formulas are presented for structures of fuzzy inference systems and artificial neural networks whose output functions are equivalent to each other and to using continuous piecewise multilinearisation for interpolation on a regular grid. This result leads to alternative interpretations, initialisation methods, and implementations of the networks.

**Introduction:** Data interpolation, fuzzy information systems (FISs), and artificial neural networks (ANNs) have very different motivations and philosophical bases, but all three are well-known to be capable of function approximation. This suggests that under certain circumstances, the three should be equivalent. When this equivalence is known to hold, then the particular features of each, such as ease of computation, clear interpretation of results, and availability of special hardware or software for implementation, can be exploited as required. Partially to this end, recently a number of papers have presented pairwise relationships among the three approaches, with the intention of exploiting the best features of each. Examples include the equivalence of certain FISs to the radial basis function ANN [1] and a relationship between spline interpolation and FISs [2]. Linear FISs have also been used [3].

We present here an FIS and ANN designed to perform continuous piecewise multilinear interpolation and thereby demonstrate the first three-way equivalence of a standard interpolation method, an FIS, and an ANN. In particular, we show that an FIS and ANN can be constructed which perform continuous piecewise multilinear interpolation of an array of scalars $u$ defined at points $\mathbf{y}$ where the $y$ are $n$-vectors which lie in a regular grid of dimensions $N_1 \times N_2 \times \cdots \times N_n$ and of numbers of elements $N = N_1N_2\ldots N_n$.

The derivations are easily performed using algebra and induction, so only the results are presented.

We denote the known value of $u(\mathbf{y})$ at the gridpoint $\mathbf{y} = [y_{i_1}, y_{i_2}, \ldots, y_{i_n}]$ by $u(i_1, i_2, \ldots, i_n)$, $i_1 = 1, 2, \ldots, N_1$, $i_2 = 1, 2, \ldots, N_2$, $\ldots$, $i_n = 1, 2, \ldots, N_n$. For notational convenience, we define the augmenting grid points $y_{0,0,\ldots,0}$ and $y_{N_1-1,0,\ldots,0}$, assume ordering such that $y_{0,0,\ldots,0} < y_{0,1,\ldots,0} < \ldots < y_{N_1-1,0,\ldots,0}$, $i_1 = 1, 2, \ldots, n$, and let the values at these new points be zero, i.e. $u(i_1, i_2, \ldots, i_n) = 0$ for any $i_n = 0$ or $i_1 = 1$. Also, the notations

$$
\begin{align*}
(1) & \quad \sum_{i_1=1}^{N_1} \sum_{i_2=1}^{N_2} \cdots \sum_{i_n=1}^{N_n} f(i_1, i_2, \ldots, i_n) \\
& \quad \sum_{i_1=1}^{N_1} \sum_{i_2=1}^{N_2} \cdots \sum_{i_n=1}^{N_n} f(i_1, i_2, \ldots, i_n) \\
& \quad \sum_{i_1=1}^{N_1} \sum_{i_2=1}^{N_2} \cdots \sum_{i_n=1}^{N_n} f(i_1, i_2, \ldots, i_n)
\end{align*}
$$

for multiple summations and $k = 0$ or 1 indicate that $k$ takes on the two values in turn will be used. We recall that a function $g(x): R \rightarrow R$ is said to be multilinear if it is linear in each component $x_i$ when all other components are fixed. A general form of such a function is

$$g(x) = a_0 + \sum_{i_1=1}^{n} a_{i_1} x_{i_1} + \sum_{i_2=1}^{n} a_{i_2} x_{i_2} + \cdots$$