CORRELATIONS AND REALISTIC INTERACTIONS IN DOUBLY CLOSED SHELL NUCLEI

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We review the latest variational calculations of the ground state properties of doubly closed shell nuclei, from $^{12}$C to $^{208}$Pb, with semirealistic and realistic two- and three-nucleon interactions. The studies are carried on within the framework of the correlated basis function theory and integral equations technique, with state dependent correlations having central and tensor components. We report results for the ground-state energy, one- and two-bodies densities and static structure functions. For $^{16}$O and $^{40}$Ca we use modern interactions and find that the accuracy of the method is comparable to that attained in nuclear matter with similar hamiltonians, giving nuclei underbound by $\sim 2$ MeV/A. The computed Coulomb sums are in complete agreement with the latest analysis of the experimental data.

1 Introduction: interaction and correlations

Our knowledge of the nuclear interaction is steadily improving because of both the ever increasing number of experimental data and the more and more sophisticated theories which are being developed to tackle the longstanding problem of an accurate description of the strongly interacting nuclear systems. The interplay between experiments and theory is clear: interactions are built which fit the data (typically from nucleon-nucleon scattering) and are then tested theoretically in more complicated structures. This approach has led to the construction of nucleon-nucleon (NN) potentials which reproduce a huge amount of NN scattering data (the latest versions fit $\sim 1800$ pp and $\sim 2500$ np data with $\chi^2 \sim 1$ and break the charge independence and charge symmetry$^{1-3}$). However, the use of these accurate, modern NN potentials in A>2 nuclei has also proved their inability to correctly describe the nuclear binding. Light nuclei are underbound and the nuclear matter saturation density is not correctly reproduced by a hamiltonian containing two-nucleon forces only. Several cures to these pathologies have been proposed, ranging from relativistic effects to extra degrees of freedom ($\Delta$’s) or to many-body forces.

If we choose to advocate the lack of binding to the presence of more-nucleon
potentials, then the first step consists in the introduction of three-nucleon interactions (TNI). The knowledge of TNI is by far less deep than that of the NN potential because of the considerably lower number of experimental data and of the higher difficulty in constructing theoretical models. However, even with these caveats, the present models of TNI provide the correct binding for \( A=3,4 \) nuclei (actually, TNI contain parameters which are fitted on light nuclei binding energy) and, more important and less obvious, bring the computed nuclear matter saturation density very close to its empirical value of \( \rho_{NM} = 0.16 \text{ fm}^{-3} \).

Following these guidelines, one can write a realistic nuclear hamiltonian in the form:

\[
H = -\frac{\hbar^2}{2m} \sum_i \nabla_i^2 + \sum_{i<j} v_{ij} + \sum_{i<j<k} v_{ijk}
\]

where \( v_{ij} \) and \( v_{ijk} \) are two- and three-nucleon potentials. The NN potential may be split into:

\[
v_{ij} = v_{ij}^{em} + v_{ij}^{\pi} + v_{ij}^R
\]

where the electromagnetic part, \( v_{ij}^{em} \) contains Coulomb, Darwin-Foldy, vacuum polarization and magnetic moments contributions; the long range part is well established from a microscopic point of view as being given by the One Pion Exchange potential (OPEP), \( v_{ij}^{\pi} \); the intermediate and short range potential, \( v_{ij}^R \), arises from the exchange of more pions and heavier mesons (\( \rho \) and \( \omega \)), however its microscopic derivation from meson exchange theory is troublesome (for the larger number of Feynman diagrams involved and the greater difficulty in the potential extraction) and a phenomenological approach is often followed, consisting in a physically plausible parametrization in terms of a reasonable number of parameters. For instance, the Argonne \( v_{18} \) NN potential\(^1\) contains 43 parameters and it is given by a sum of 18 terms

\[
v_{ij} = \sum_{p=1,18} v_p (r_{ij}) O_{ij}^p,
\]

where the first 14 terms are isoscalar, with

\[
O_{ij}^{p=1,14} = [1, \sigma_i \cdot \sigma_j, S_{ij}, L \cdot S, L^2, L^2 \sigma_i \cdot \sigma_j, (L \cdot S)^2] \otimes [1, \tau_i \cdot \tau_j],
\]

and the remaining isovector and isotensor components are:

\[
O_{ij}^{p=15,18} = [1, \sigma_i \cdot \sigma_j, S_{ij}, 3\tau_i \tau_j - 1, \tau_i \cdot \tau_j], 2[\tau_i \tau_j + \tau_j \tau_i].
\]

Two-pion exchange gives a large part of the attraction of the three-nucleon potential, whereas the remainder is mostly phenomenological. Recently, attempts are being made to evaluate microscopically other pieces, as the three-pion two-Delta diagram.\(^4\) Popular forms of the TNI are the Urbana forces,\(^5\) given by

\[
v_{ijk} = v_{ijk}^{2\pi} + v_{ijk}^R
\]

The two-pion Fujita-Miyazawa\(^6\) term, \( v_{ijk}^{2\pi} \), is

\[
v_{ijk}^{2\pi} = \sum_{cyc} A_2 \{X_{ij}, X_{ik}\}\{\tau_i \cdot \tau_j, \tau_i \cdot \tau_k\} + C_2 \{X_{ij}, X_{ik}\}\{\tau_i \cdot \tau_j, \tau_i \cdot \tau_k\},
\]

\[
(p_{ij})^{4} - \frac{\hbar^2}{2m} \sum_i \nabla_i^2 + \sum_{i<j} v_{ij} + \sum_{i<j<k} v_{ijk}
\]

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\]

\[
(p_{ij})^{4} - \frac{\hbar^2}{2m} \sum_i \nabla_i^2 + \sum_{i<j} v_{ij} + \sum_{i<j<k} v_{ijk}
\]
with

$$X_{ij} = Y_\pi(r_{ij})\sigma_i \cdot \sigma_j + T_\pi(r_{ij})S_{ij},$$

(8)

$Y_\pi(r)$ and $T_\pi(r)$ being the Yukawa and tensor Yukawa functions. The short range part, $u^{2\pi}_{ijk}$, is written as

$$u^{2\pi}_{ijk} = \sum_{cycl} U_0 T^2_\pi(r_{ij})T^2_\pi(r_{ik}) .$$

(9)

$A_{2\pi}, C_{2\pi}$ e $U_0$ are chosen to reproduce the binding of $A=3,4$ nuclei.

Once a realistic hamiltonian has been built, the task of the theorist is to address the ground state Schrödinger equation, $H\Psi_0 = E_0\Psi_0$, to solve it or to find good approximations to its solutions. The exact solution is now possible in light nuclei by a variety of techniques: Green's Function Monte Carlo\textsuperscript{7} for $3 \leq A \leq 8$, Faddeev and Faddeev-Yakubovsky\textsuperscript{8} and Correlated Hyperspherical expansions (CHE).\textsuperscript{9} GFMC has been pushed to the highest $A$-value, whereas the other approaches have been used mainly for $A = 3,4$. However, they are more flexible in the sense that both Faddeev and Correlated Hyperspherical expansion methods are more easily extendible to the study of three- and four-body scattering reactions. In fact, several few-body reactions of relevant astrophysical interest have been accurately analyzed by CHE. For the remaining part of the nuclear table, including nuclear matter, methods to exactly solve the Schroedinger equation are still to come, although a very promising approach, based on the Hubbard-Stratonovich transformation, is currently under development.\textsuperscript{10} Hence, alternative ways to gather information on the many-body wave function must be devised. The Brueckner theory has brought standard perturbative theories to an extremely high level of sophistication;\textsuperscript{11} the Coupled Cluster (CC) method has been used to build correlations into the wave function and is, by now, a standard technique in many-body physics;\textsuperscript{12} the variational principle has provided a powerful recipe to construct accurate wave functions.

A direct consequence of the strong nuclear interaction is the failure of the mean field (MF) approach in the description of the nucleus. Striking evidence of this fact is provided by $(e,e')p$ experiments\textsuperscript{13} where a clear signature of a depletion of the occupation probability with respect to unity for states below the Fermi level, $\epsilon_F$, (as required by MF theories) is present, together with a non zero occupation probability for states above $\epsilon_F$. This behavior may be explained in terms of short range dynamical correlations (in contrast with statistical correlations, due to the antisymmetry) generated in the wave functions by the interaction and which can be hardly described by standard perturbation methods based on a non interacting basis. For instance, Brueckner theory must sum infinite numbers of ladder diagrams in order to deal efficiently with the nuclear potential.

In the Correlated Basis Function (CBF) theory the non perturbative correlation effects are directly embedded into the basis functions. This property makes the theory a powerful tool to investigate many-body interacting systems in several fields of physics, as liquid Helium, electronic structures (both in the forms of electron fluids and lattices) and both finite nuclei and infinite nuclear matter. The flexibility of the CBF approach results in a realistic description not only of the ground state (energy, momentum distribution, distribution functions and so on) but also of dynamical (cross sections) quantities.
A set of correlated basis wave functions, \( \Psi_n(1,2...A) \), may be built by applying a many-body correlation operator, \( F(1,2...A) \), to the model basis functions, \( \Phi_n(1,2...A) \),

\[
\Psi_n(1,2...A) = F(1,2...A)\Phi_n(1,2...A) ,
\]

where the operator \( F \) is intended to take care of the dynamical correlations, whereas the model wave functions, \( \Phi_n \), include antisymmetrization effects and, possibly, long range correlations due to collective excitations (as BCS type states or surface effects). A perturbative theory may be developed in terms of the correlated states (10), having the nice property of a rapid convergence and the counterindication of a large difficulty in computing the matrix elements. The zeroth order of this theory is often referred to as the variational level of CBF. In fact, the variational principle is used to fix the correlation operator by minimizing the ground state energy, \( E_0^\psi = \langle \Psi_0 | H | \Psi_0 \rangle / \langle \Psi_0 | \Psi_0 \rangle \).

The choice of the correlation operator depends, to a large extent, on the interaction. A form of \( F(1,2...A) \) that has shown to be suitable to nuclear systems is

\[
F(1,2...A) = S\left\{ \prod_{i<j=1,A} F_{ij} \right\} ,
\]

i.e. a symmetrized product of two-body correlation operators, \( F_{ij} \). The model states, \( \Phi_n(1,2...A) \), are Slater determinants of single particle wave functions, \( \phi_\alpha(i) \), obtained by some MF potential (simple plane waves in infinite, homogeneous matter). \( F_{ij} \) is chosen, consistently with the interaction, to be given by

\[
F_{ij} = \sum_{p=1,8} f^p(r_{ij})O^p_{ij} ,
\]

where the sum runs up to the spin-orbit components in Eq. (4). If the \( p \geq 2 \) components are disregarded, the Jastrow scalar correlation is recovered. The free minimization of \( E_0^\psi \) would provide the best choice for the correlation functions \( f^p(r) \). However, this is a prohibitive task in nuclear systems, so the correlation functions are usually parametrized and the parameters are fixed by minimizing \( E_0^\psi \). An additional minimization may be performed on the parameters of the MF model wave function.

2 Cluster expansions and Fermi hypernetted chain equations

As already mentioned in the previous section, computing the matrix elements in the CBF approach represents the greatest difficulty in its application. In principle one could use Monte Carlo (MC) based algorithms to sample the occurring multi-dimensional integrals. For simple interactions and correlations this can be done even for very large numbers of particles. In such a case, the \( A \rightarrow \infty \) limit can also be studied by means of appropriate conditions at the borders of the simulation box. Liquid Helium, both as a fluid and as droplets, has been the subject of MC investigations. However, the strong state dependence of the nuclear interaction prevent the use of MC techniques in medium-heavy nuclei, as well as in nuclear
matter. The $^{16}$O ground state was studied within CBF by the cluster Monte Carlo (CMC) method,\textsuperscript{14} where the Jastrow correlations contribution is exactly treated by MC sampling, and that from the remaining operatorial components is approximated by considering (via MC) up to four- or five-body cluster terms.

An alternative approach is provided by cluster expansions. The expectation values of operators are written in terms of $n$-body densities,

$$\rho_1(1) = \langle \sum_r \delta(r_i - r_1) \rangle ,$$

$$\rho_2^{(p)}(1, 2) = \langle \sum_{i \neq j} \delta(r_i - r_1)\delta(r_j - r_2) O_{ij}^p \rangle ,$$

and related quantities, as the density matrices. The densities are then cluster expanded in terms of dynamical, $h(r) = [f^1(r)]^2 - 1$ and $f^1(r) f^{p \geq 2}(r)$, and statistical, $\rho_0(i, j) = \sum_\alpha \phi_\alpha^*(i) \phi_\alpha(j)$, correlations. The expansion is linked, in the sense that disconnected diagrams, given by the product of two or more pieces not connected among each other by any kind of correlation and coming from the expansion of the numerator, cancel exactly with those generated by the expansion of the denominator. However, the cluster expansion is not irreducible, i.e. the surviving connected diagrams bear vertex corrections (except for the Jastrow correlation in the $A \to \infty$ case, where the vertex correction is simply the density).

The cluster diagrams are then classified into: Nodal, N, where all the paths between the external points (for instance, points 1 and 2 for the two-body density of Eq. (14)) go through the same internal point, or node; Composite, C, given by the product of two or more nodals; the remaining diagrams belong to the Elementary class, E.

Fermi Hypernetted Chain (FHNC) integral equations\textsuperscript{15} allow for the exact summation of the nodal and composite diagrams, in the Jastrow case, once the sum of the elementary diagrams, $E(1, 2)$, is given. So, the function $E(1, 2)$ is actually an input for solving the FHNC equations. Unfortunately, no exact way of computing this function is known and one must use approximations. The simplest one is the FHNC/0 truncation, where the choice $E = 0$ is made. This seemingly crude approximation is however appropriate to nuclear systems, where the density is not very high. In denser liquid Helium, a more accurate treatment of the elementary diagrams is needed.

For state dependent correlations, a complete FHNC summation is possible only for the Jastrow part. In fact, different orderings of the operators in the symmetrized product (11) may give the same cluster contribution and a scheme to correctly keep track of all of them has not been devised so far. However, partial classes of diagrams containing operatorial correlations may be exactly summed by the single operator chain approximation (FHNC/SOC).\textsuperscript{16} The FHNC/SOC integral equations sum all the nodal diagrams with only one operatorial correlation per internal side, besides all the Jastrow correlated clusters. The accuracy of the FHNC/SOC approximation has been set to $\sim 1$ MeV/$A$ in nuclear matter at saturation density by computing its leading corrections.\textsuperscript{17} The FHNC/SOC scheme has been recently extended to doubly closed shell nuclei in $ls$ coupling.\textsuperscript{18}

A way to control the approximations is provided by the density sum rules. In
Table 1. One- and two-body density sum rules in $^{16}$O and $^{40}$Ca for Jastrow and operatorial correlations.

<table>
<thead>
<tr>
<th></th>
<th>$S_1$</th>
<th>$S_2$</th>
<th>$S_\tau$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^{16}$O</td>
<td>$f_1$</td>
<td>16.00</td>
<td>1.00</td>
</tr>
<tr>
<td></td>
<td>$f_6$</td>
<td>16.01</td>
<td>1.05</td>
</tr>
<tr>
<td>$^{40}$Ca</td>
<td>$f_1$</td>
<td>40.00</td>
<td>1.00</td>
</tr>
<tr>
<td></td>
<td>$f_6$</td>
<td>39.86</td>
<td>1.09</td>
</tr>
</tbody>
</table>

The exact densities satisfy the following normalizations:

\[ \int dr_1 \rho_1(1) = A \quad , \quad (15) \]

\[ \frac{1}{A(A-1)} \int dr_1 \int dr_2 \rho_1^{(1)}(1,2) = 1 \quad , \quad (16) \]

\[ \frac{1}{3A} \int dr_1 \int dr_2 \rho_2^{(r)}(1,2) = -1 \quad . \quad (17) \]

The last equality holds for isospin saturated systems. Deviations from the sum rules are a powerful check for the approximations made in the solution of the FHNC equations, both in the treatment of the elementary diagrams and of the operatorial correlations.

As a matter of fact, FHNC/SOC satisfies the normalizations of the densities to a high degree of accuracy, as it is shown in Table 1. The Table gives the one-body ($S_1$, Eq. (15)) and central and isospin two-body ($S_2$ and $S_\tau$, eqs. (16,17)) density normalizations for Jastrow ($f_1$) and operatorial ($f_6$) correlations in $^{16}$O and $^{40}$Ca. The $f_6$ model does not contain spin-orbit components but it does have the tensor ones. The Jastrow model satisfies the sum rules almost exactly, whereas the SOC approximation produces discrepancies of $\sim 9\%$ in the worst case.$^{18}$

3 Energies, densities and structure functions

In a series of papers$^{19–21}$ we extended the FHNC formalism to doubly closed shell nuclei, both in $ls$ and $jj$ coupling scheme, described by Jastrow correlated wave functions. In those papers model interactions were used, such as the semirealistic Afnan and Tang (S3) one.$^{22}$ The original S3 potential is purely central, without tensor components, and reproduces the NN $s$-waves scattering data up to $\sim 60$ MeV. Its modified version$^{23}$ has been supplemented in the odd channels of the same repulsion as in the even ones and gives a nuclear matter binding curve close to the current realistic potentials. The correlations we adopted contained a $\tau_z$ dependence, allowing for distinguishing between the different nucleon pairs ($f_{nn} \neq f_{pp} \neq f_{np}$). The ground state energies for the $\tau_z$-dependent correlations are shown in Table 2 together with the experimental values. The correlations have been fixed by solving the Euler equations which result from the minimization of the expectation value of the hamiltonian at the second order of the cluster expansion. Given the poor quality of the interaction, the comparison with experiment is not really meaningful, even
Table 2. Ground state energies for doubly closed shell nuclei with the S3 potential and Jastrow correlated wave functions. The EUL and ACA lines give the energies with $\tau_\alpha$ dependent and independent correlations, respectively. Energies in MeV.

<table>
<thead>
<tr>
<th></th>
<th>$\langle H \rangle$</th>
<th>E/A</th>
<th>(E/A)$_{\text{exp}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^{12}$C</td>
<td>EUL</td>
<td>-33.2</td>
<td>-3.84</td>
</tr>
<tr>
<td></td>
<td>ACA</td>
<td>-15.5</td>
<td>-2.36</td>
</tr>
<tr>
<td>$^{16}$O</td>
<td>EUL</td>
<td>-119.7</td>
<td>-8.20</td>
</tr>
<tr>
<td></td>
<td>ACA</td>
<td>-82.8</td>
<td>-5.89</td>
</tr>
<tr>
<td>$^{40}$Ca</td>
<td>EUL</td>
<td>-381.2</td>
<td>-9.78</td>
</tr>
<tr>
<td></td>
<td>ACA</td>
<td>-262.1</td>
<td>-6.81</td>
</tr>
<tr>
<td>$^{48}$Ca</td>
<td>EUL</td>
<td>-394.9</td>
<td>-8.43</td>
</tr>
<tr>
<td></td>
<td>ACA</td>
<td>-272.3</td>
<td>-5.87</td>
</tr>
<tr>
<td>$^{208}$Pb</td>
<td>EUL</td>
<td>-1761.4</td>
<td>-8.50</td>
</tr>
<tr>
<td></td>
<td>ACA</td>
<td>-1056.3</td>
<td>-5.11</td>
</tr>
</tbody>
</table>

Table 3. $^{16}$O and $^{40}$Ca ground state energies per nucleon and radii for the Argonne $\nu_{14}$ + Urbana VII and the Argonne $\nu_8'$ + Urbana IX models with the FHNC/SOC, Cluster Monte Carlo (CMC) and Coupled Cluster (CC) methods.

<table>
<thead>
<tr>
<th></th>
<th>E/A (MeV)</th>
<th>rms (fm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^{16}$O</td>
<td>FHNC</td>
<td>-5.97</td>
</tr>
<tr>
<td></td>
<td>CMC</td>
<td>-6.30</td>
</tr>
<tr>
<td></td>
<td>CC</td>
<td>-6.10</td>
</tr>
<tr>
<td>$^{40}$Ca</td>
<td>FHNC</td>
<td>-5.41</td>
</tr>
<tr>
<td></td>
<td>expt</td>
<td>-7.98</td>
</tr>
<tr>
<td></td>
<td>FHNC</td>
<td>-6.64</td>
</tr>
<tr>
<td></td>
<td>expt</td>
<td>-8.55</td>
</tr>
</tbody>
</table>

if not completely unsatisfactory. However, it is interesting to note a saturation trend of the binding energy along the mass number. The Table also gives the energies obtained by an isospin independent correlation, identical for all the pairs (Average Correlation Approximation, ACA). The poorer variational quality of the ACA correlation produces higher energies than the isospin dependent model. The density normalizations are always satisfied with an accuracy greater than percent for the calculations of the Table.

FHNC/SOC has been pushed to deal with realistic potentials and state dependent, non-central correlations in $^{16}$O and $^{40}$Ca in Refs. 18, 24. In the last work a complete hamiltonian of the form given in Eq. (1) with the inclusion of the momentum dependent parts of the NN interaction and of the three-nucleon potential, as well as of the Coulomb term, was adopted. Table 3 gives the g.s. energies per nucleon for two models: Argonne $\nu_{14}$ + Urbana VII$^{25,26}$ (A14+UVII) and Argonne $\nu_8'$ + Urbana IX$^{7}$ (A8'+UIX). The correlated wave function features a $f_0$ nuclear matter correlation and single particle wave functions generated by a Woods-Saxon mean field potential. In the A14+UVII case, the $^{16}$O FHNC/SOC energies are compared with the CMC results of Ref. 14, obtained with a similar wave function, and with the Coupled Cluster (CC) ones of Ref. 27. The r.m.s. radius is also given.
The variational A14+UVII energies are close to the CC estimates, while the discrepancies with CMC are due to the approximations in the FHNC/SOC scheme and to small differences in the correlation. Actually, our nuclear matter Euler equation does not contain the spin-orbit potential, whereas that used for the CMC correlations does. This fact produces small variations, which mostly affect the kinetic energy. FHNC and CMC give similar radii, both of them smaller than CC. Explicit three-body correlations, not given by the product of two-body ones, were found to provide $\sim 0.85$ MeV/A extra binding$^{14}$ in the A14+UVII case.

The charge densities, $\rho_{c}(r_{1})$, and the two-body distribution functions, $\rho_{2}(r_{12})$, computed with the A8'+UIX wave function are shown in Fig. 1. The CBF $\rho_{c}(r_{1})$ (dashed lines) are folded with the proton form factor and compared with the experimental data (solid lines). $\rho_{2}(r_{12})$ is defined as

$$
\rho_{2}(r_{12}) = \frac{1}{A} \int d^{3} R_{12} \rho_{2}^{(1)}(r_{1}, r_{2}) ,
$$

(18)

where $R_{12}$ is the center of mass coordinate. Short range correlations strongly affect $\rho_{2}(r_{12})$ (solid lines) at small internucleon distances, where the NN repulsion heavily depletes the distribution functions with respect to the independent particle model (IPM) estimates (dashed lines).

Two-body densities are also of great interest since they allow for analyzing several integrated nuclear cross sections. In fact, the responses of a nucleus to an external probe, described by an operator $O_{X}$, can be expressed in terms of the dynamical structure functions, $S_{X}(q, \omega)$, whose non energy weighted sum gives the static structure function (SSF), $S_{X}(q)$. In turn, the SSF are given by expectation values of appropriate combinations of one- and two-body densities. For instance, the electromagnetic longitudinal response, $S_{L}(q, \omega)$, as measured in electron-nucleus
experiments, is mostly due to charge fluctuations. The corresponding SSF, $S_L(q)$ (or Coulomb sum), is given by

$$S_L(q) = 1 + \frac{1}{Z} \int d^3r_1 \int d^3r_2 \ e^{i\mathbf{q} \cdot \mathbf{r}_{12}} [\rho_{pp}(r_1, r_2) - \rho_c(r_1)\rho_c(r_2)] \ ,$$

(19)

where $\rho_{pp}(r_1, r_2)$ is the proton-proton two-body density.

In Fig. 2 we compare the A8'+UIX CBF Coulomb sums with those extracted from the world data on inclusive quasi-elastic electron scattering experiments in $^{12}$C, $^{40}$Ca, and $^{56}$Fe. The figure also shows the nuclear matter Coulomb sum for the A14+UVII model from Ref. 29. The agreement of the nuclei theoretical SSF with the latest experimental data, which properly take into account the large energy tail of the response, is complete, whereas the nuclear matter shows some discrepancies at the lowest $q$ values where finite size effects appear to be still relevant.

4 Conclusions

Appreciable progress in the microscopic study of the ground state structure of medium-heavy, doubly closed shell nuclei has been made in the latest years. Going beyond the simple mean field picture and using realistic hamiltonians seems now to be within reach of many-body theories. In this context, the variational approach and the correlated basis function theory are among the most promising tools. Fermi hypernetted chain and single operator chain techniques appear to deal with the complexity of the interaction and of the correlation with the same degree of accuracy that has been achieved in nuclear matter. This opens the field both to the microscopic study of dynamical quantities, as the nuclei cross sections, and to the extension of the variational method to other interesting systems, as, for instance, hypernuclei.
References