We point out that the cross section for the process $e^+e^- \rightarrow e^+e^-\tau^+\tau^-$ at CERN LEP is large enough to allow for a study of the anomalous electromagnetic couplings of the $\tau$ lepton. We show that competitive bounds for the magnetic and electric dipole moments can be obtained using the data taken from 1992 to 1994. Finally, we show how these bounds can be improved at LEP II.

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The results obtained by the four experiments at the CERN $e^+e^-$ collider LEP since they started taking data have allowed for precision studies of $Z$-boson physics that have confirmed the standard model predictions at the level of a few per mil [1]. This was, certainly, the main purpose of LEP and, up to now, all the efforts have been focused in this direction. However, LEP has other physics potentials that should be exploited. In particular, two-photon processes have large cross sections that might be only one order of magnitude smaller than the corresponding cross sections at the $Z$ peak. A study of the photon structure function in $\mu$ pair and $\tau$ pair production in two photon collisions has been performed by the OPAL Collaboration [2]. In this Brief Report we want to discuss the possibility of studying the electromagnetic anomalous couplings of the $\tau$ lepton via the process

$$e^+e^- \rightarrow e^+e^-\tau^+\tau^-$$

at LEP with the data already in tape. We will also give some estimations of the sensitivity of LEP II to these anomalous couplings. A similar analysis for heavy ion colliders has already been presented in Ref. [3].

The $\tau$-lepton couplings can offer an interesting window to physics beyond the standard model. Although the bounds one can expect to obtain for these couplings will never be as strong as the ones already available for the other leptons [4], they may be theoretically more relevant. Very often the new theories or models predict values for the anomalous couplings that are enhanced for higher values of the particle mass, making the $\tau$ the ideal candidate among the leptons to observe these new couplings.

Two-photon processes offer several advantages to study the electromagnetic $\tau$ couplings at LEP. First of all, this process is free from the uncertainties originated by possible anomalous $Z\tau\tau$ couplings [5]. Moreover, since photons are almost real and the invariant mass of the $\tau$ pair is very small, we expect the effects of unknown form factors to be negligible. Finally, the relevant kinematical region in the two-photon process is complementary to the one at DESY $e^+e^-$ collider PETRA and LEP (in processes mediated by the $Z$), where the present bounds have been obtained.

The magnetic and electric dipole moments of the tau can be easily introduced adding two terms to the standard model $\tau\gamma$ vertex [6]:

$$-ie\epsilon_\mu(q') \left( F_1(q^2)\gamma^\mu + iF_2(q^2)\sigma^{\mu\nu} \frac{q_\nu}{2m_\tau} + F_3(q^2)\gamma_5\sigma^{\mu\nu} \frac{q_\nu}{2m_\tau} \right) u(p)\epsilon_\mu(q) ,$$

(2)

where $\epsilon_\mu(q)$ is the polarization vector of the photon with momentum $q$, $F_1(q^2)$ is related to the electric charge, $F_2(q^2)$ and $F_3(q^2)$ are the form factors related to the magnetic and electric dipole moments, respectively, through

$$\mu_\tau = \frac{e(F_1(0) + F_2(0))}{2m_\tau} \quad ; \quad d_\tau = -\frac{eF_3(0)}{2m_\tau} .$$

(3)

In the standard model at the tree level, $F_1(q^2) = 1$ and $F_2(q^2) = F_3(q^2) = 0$. It should be noted that the $F_2$ term behaves under $C$ and $P$ like the standard model one, while the $F_3$ term violates $CP$. We will restrict our discussion to tree level. Standard model radiative corrections can be calculated and the data can be corrected for their effects.

Bounds on the values of $F_2$ and $d_\tau$ at 1$\sigma$ have been obtained from the study of the $\tau$ angular distribution in $e^+e^- \rightarrow \tau^+\tau^-$ at PETRA:

$$|F_2| \leq 0.014 \quad [7],$$

$$|d_\tau| \leq 1.4 \times 10^{-16} \mathrm{e\ cm} \quad [8].$$

(4)

These bounds, however, neglect the effects of the form factors from $q^2 = 0$ to $\sim 1.5 \times 10^3$ GeV$^2$, where the measurements were taken. Grilfos and Mendoza [9] proposed to look for deviations from the standard model predictions in the cross section for $e^+e^- \rightarrow \tau^+\tau^-\gamma$. Using this method and assuming a standard $Z\tau\tau$ coupling, the bounds [10]
\[ |F_2(0)| \leq 0.072, \]
\[ |d_\tau| \leq 4 \times 10^{-16} \text{ cm} \quad (5) \]

at 95% C.L. have been obtained. Recently, much more stringent bounds have been obtained by Escribano and Massó [11]:
\[ |F_2| \leq 0.0062, \quad |d_\tau| \leq 3.4 \times 10^{-17} \text{ cm} \quad (6) \]

at 1σ level. These bounds arise from the good agreement between the standard model predictions and the experimental data obtained at LEP for the decay width \( \Gamma(Z \to \tau^+ \tau^-) \). What it is actually measured, thus, is the coupling \( \varepsilon\gamma\tau \) that can be related to the one we are concerned with, \( \gamma\tau\tau \), requiring an SU(2)\( \times \)U(1) gauge invariance. Therefore, the bounds in Eq. (6) have more theoretical input than the previous ones and the ones that can be obtained in photon-photon collisions.

The value of the total cross section in the standard model, using the equivalent photon approximation [12], is
\[ \sigma(e^+e^- \to e^+e^-\tau^+\tau^-) = 0.303 \text{ nb}. \quad (7) \]

Since the collected integrated luminosity from 1992 to 1994 is \( L = 100 \text{ pb}^{-1} \) per experiment, the total number of \( \tau \) pairs produced during these years in no-tag two-photon collisions is 31,000. Because of the effective \( \gamma\gamma \) luminosity, the invariant mass of these \( \tau \) pairs is strongly peaked near threshold. This has two important consequences. First of all, it allows a clear cut experimental separation from the \( \tau \tau \) events produced in \( Z \) decays. Indeed, a cut such as \( M_{\tau\tau} \leq 20 \text{ GeV} \) would only discard a 1.5% of the two-photon events. Second, it is well known that the introduction of the anomalous couplings leads to unitarity violations that do not appear in the standard model because of gauge cancellations. Thus, any realistic theory that produces nonzero values for the couplings \( F_2 \) and \( F_3 \), will also produce some modulating form factors that will dampen the contributions from these couplings at large energies. The effects of these form factors, however, are expected to be small near threshold, so we will neglect them.

The dependence of the total cross section with \( F_2 \) at LEP, assuming \( F_3 = 0 \), is shown in Fig. 1. The cross section is a quartic polynomial in \( F_2 \), but the smallness of the relevant values of \( F_2 \) allows for a very good linear approximation as can be seen in the figure. The dotted line corresponds to the central value of the standard model cross section [Eq. (7)] and the dashed line shows the statistical error per experiment assuming a \( \tau \) pair detection efficiency \( \epsilon = 0.01 \). With these assumptions the bound
\[ |F_2(0)| \leq 0.02 \quad (8) \]

can be obtained. This bound is similar to the previous ones but it is complementary to them because it has been obtained in a different kinematical region and it is free from the theoretical assumption that led to the result in Eq. (6). We can exploit the approximate linear dependence on \( F_2 \) of the cross section to obtain a simple expression for the bound in \( F_2 \) as a function of the total integrated luminosity and the efficiency in the \( \tau \) pair detection:
\[ |F_2(0)| \leq \frac{0.02 \text{ pb}^{-1/2}}{\sqrt{Le}}, \quad (9) \]

where \( L \) is expressed in \( \text{ pb}^{-1} \), \( \epsilon \) is the efficiency in the detection of \( \tau \) pairs, and \( n \) is the number of standard deviations required.

The behavior of the cross section as a function of \( d_\tau \) is shown in Fig. 2 with the same parameters and notation as in the previous figure. From the figure we can read

\[ |F_2| \leq 0.018 \text{ for } d_\tau = 4 \times 10^{-16} \text{ cm}. \]

FIG. 1. Total cross section for the process \( e^+e^- \to e^+e^+\tau^+\tau^- \) as a function of the anomalous magnetic moment, \( F_2 \) (solid line), at LEP. The dotted line stands for the standard model prediction and the dashed line shows the 1σ statistical error assuming an integrated luminosity of 100 \( \text{ pb}^{-1} \) and a 1% efficiency in the detection of the produced \( \tau \) pairs.

FIG. 2. Total cross section for the process \( e^+e^- \to e^+e^+\tau^+\tau^- \) as a function of \( d_\tau \) with the same parameters and notation as in the previous figure.
the bound for the electric dipole moment of the \(\tau\) that can be reached at LEP:

\[
|d_\tau| \leq 4.8 \times 10^{-10} \text{ e cm.} \tag{10}
\]

This is a factor of 3 worse than the bound from PETRA, although the comment about the complementarity of the bounds apply also here. The main difference with the magnetic moment case is that, due to the \(CP\)-violating nature of the electric dipole moment term, the interference with the standard model cancels. The dependence of the cross section with \(F_3\) is, thus, quadratic in a very good approximation, hence the comparatively lower sensitivity of our process to the electric dipole moment than to the magnetic one. In any case, we can also obtain a simple expression for the bound in terms of the integrated luminosity and the efficiency as in Eq. (9):

\[
d^2 \leq \frac{2.3 \times 10^{-31} \text{ pb}^{-1/2}}{\sqrt{\mathcal{L} \epsilon}} (\text{e cm})^2. \tag{11}
\]

The previous bounds cannot be improved studying the \(p_T\) distribution of the \(\tau\) pairs. The only difference in the symmetry properties between the standard model and the magnetic dipole moment term is the helicity change introduced by the latter. However, since the \(\tau\)'s are produced near threshold, where the mass is of the same order as the photon-photon center-of-mass energy, this difference does not generate a measurable effect in the \(p_T\) distribution.

Regarding the electric dipole moment term, the cancellation of the interference with the standard model hides its \(CP\)-violating nature, so the differences introduced in the \(p_T\) distribution are again negligible.

A different way to look for effects of the electric dipole moment is to study \(CP\)-odd observables. Since the initial two-photon state, when adding over all possible initial configurations, transforms into itself under \(CP\), such observables are a clear signature for \(CP\) violation and are expected to cancel at tree level in the standard model. In the photon-photon center-of-mass system the \(\tau\) and \(\bar{\tau}\) momenta remain unchanged under \(CP\) transformations, while their spins are interchanged. So, in order to isolate a \(CP\)-violating term, we have to study the production of polarized \(\tau^a\). Another, more technical, argument in this direction is that the \(CP\)-violating term contains the Levy-Civita tensor \(\epsilon_{\mu\nu\rho\delta}\), so we need four independent vectors to contract with this tensor, requiring the introduction of the final state polarization vectors.

A complete list of the observables involving the momenta and polarization of the \(\tau\) and \(\bar{\tau}\) was given in Ref. [14]. These observables require the measurements of the momenta \(P_\tau\) and \(P_{\bar{\tau}}\) and their polarization analysis via their decay products. However, in two-photon collisions, due to the lack of knowledge of the center-of-mass energy and the rapidity in each event and the presence of the neutrinos in the \(\tau\)-decay products, the momenta of the \(\tau\) and \(\bar{\tau}\) cannot be reconstructed.

We have studied exclusive, one-prong decay modes of the \(\tau\); i.e.,

\[
e^+e^- \rightarrow e^+e^-\tau^+\tau^- \rightarrow e^+e^-a^+b^-\nu_\tau\bar{\nu}_\tau X, \tag{12}
\]

where \(a^+, b^-\) are the charged particles in the \(\tau^+, \tau^-\) decay products, respectively, and \(X\) might be any other neutral particle. The observables will be constructed only in terms of the momenta of the final, observed charged particles. The polarization-analyzing power of the decay channels is characterized by a constant \(\alpha_n\) (with \(|\alpha_n| \leq 1\)) which is shown in Table I for the most relevant decay channels, together with their respective branching ratios [15]. The most sensitive observable turns out to be

\[
\hat{T}_{33} = (\hat{q}^+ \cdot \hat{q}^-)_3 (\hat{q}^+ \times \hat{q}^-)_3 \tag{13}
\]

where \(\hat{q}^\pm\) is the unit vector in the direction of the corresponding final state charged particle. Since the \(\tau\) couplings to the photon are vectorlike, the expectation values of this observable for the channel in Eq. (12) is related to the pion-pion channel through

\[
<\hat{T}_{33} >_{\tau^+\tau^-} = \alpha_n <\hat{T}_{33} >_{\pi^+\pi^-}. \tag{14}
\]

From the values at Table I it is easy to see that the most promising channel from the theoretical point of view to look for \(CP\)-violating effects in this observable is \(e^+e^- \rightarrow e^+e^-\pi^+\pi^-\nu_\tau\bar{\nu}_\tau\). Using this channel the bound that can be obtained is

\[
|d_\tau| \leq 2.3 \times 10^{-15} \text{ e cm.} \tag{15}
\]

This is one order of magnitude higher than the bound that can be obtained from the total cross section because of the poor statistics because of the small \(\tau \rightarrow \pi \nu\) branching ratio. In contrast to the bound in Eq. (11), this bound scales with the number of standard deviations required.

At LEP II the bounds obtained at LEP can be improved because of two reasons. First of all, contrary to the behavior of the annihilation process, the cross section for (1) grows with the \(e^+e^-\) center-of-mass energy. Secondly, the design luminosity of LEP II is larger than the one of LEP. In summary, the number of \(\tau\) pairs that can be produced in one year at each LEP II experiment with a center-of-mass energy \(\sqrt{s} = 200\) GeV and an integrated luminosity and the efficiency as in Eq. (9):

\[
d^2 \leq \frac{2.3 \times 10^{-31} \text{ pb}^{-1/2}}{\sqrt{\mathcal{L} \epsilon}} (\text{e cm})^2. \tag{11}
\]
luminosity $L = 300 \text{ pb}^{-1}$ is 141 000, thus, reducing the statistical error a factor $\sim 2.2$ with respect to the LEP error. The bounds for the magnetic and electric dipole moments that can be obtained from the total cross section, assuming again an efficiency $\epsilon = 0.01$, are

$$|F_2(0)| \leq 7.9 \times 10^{-3},$$

$$|d_\tau| \leq 2.8 \times 10^{-10} \text{ e cm}. \quad (16)$$

The situation for the $CP$-odd observables is similar to the one we found at LEP. The bound on $d_\tau$ that can be obtained is sensibly worse than the one in Eq. (16), $d_\tau \leq 10^{-10} \text{ e cm}$. 

In summary, we have discussed the possibility of using the data that is already on tape at LEP to look for effects of anomalous magnetic and electric dipole moments in two-photon processes. The bounds that can be obtained for the magnetic dipole moment are similar to the present ones, while for the electric dipole moment the bounds are a factor of 3 worse than the ones from PETRA. However, we should stress that our bounds are obtained with real photons and very near threshold. Hence, the kinematical region and the obtained bounds are complementary to the present ones obtained in $e^+e^-$ annihilation at $\sqrt{s} \sim 40 \text{ GeV}$. Moreover, the process we propose provides a direct way to measure the electromagnetic couplings of the $\tau$ lepton, without the requirement of additional theoretical assumptions. Because of the impossibility of reconstructing the $\tau$'s in this process, $CP$-odd observables have to be constructed using only the momenta of the $\tau$-decay products. The statistics is, thus, much poorer than for the total cross section measurement and the sensitivity to the electric dipole moment is lower using these observables than the one measuring the total cross section.

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